

ASYMMETRIC COUNTRY CUSTOMS UNION FORMATION

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Abstract

Using an n -heterogeneous-country Cournot intraindustry model with three dimensions of heterogeneity considered: i.e., marginal cost of production, market size, and industry concentration, the current paper shows that even under open regionalism regime, in contrast with Yi (1996), global free trade may not be attained via the expansion of customs union. The paper also characterizes the endogenous custom union formation path. When countries are farsightedly rational, customs union can serve as a stepping stone for multilateral liberalization.

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1 Introduction

Preferential Trade Areas (PTAs), or Regional Trade Agreements (RTAs) including Free Trade Areas (FTA) and Customs Unions (CU) experience rapid proliferation in recent years. According to the report of the World Trade Organization (WTO), 421 PTAs have

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been notified to GATT or WTO till December, 2008, and among them 230 agreements are currently in force. The official website of WTO on regionalism provides us with a concise yet excellent introduction to this topic.

“One of the most frequently asked questions is whether these regional groups help or hinder the WTO’s multilateral trading system...The WTO agreements recognize that regional arrangements and closer economic integration can benefit countries. It also recognizes that under some circumstances regional trading arrangements could hurt the trade interests of other countries. Normally, setting up a customs union or free trade area would violate the WTO’s principle of equal treatment for all trading partners (“most-favoured-nation”). But GATT’s Article 24 allows regional trading arrangements to be set up as a special exception, provided certain strict criteria are met.” (http://www.wto.org/english/thewto_e/whatis_e/tif_e/bey1_e.htm, February 20, 2009)

Whether Preferential Trade Agreements, such as Customs Unions, “building stones” or “stumbling blocks” for global free trade is the central question in this topic with heated debate in recent years. Jacob Viner (1950)’s pioneering work on customs unions introduces the Vinerian concepts of “trade creation” and “trade diversion” and shows that the ROW and even the member countries can be worse off. A positive result on CU in which the welfare of the union member countries will improve and that of ROW does not fall (due to CU formation), was stated independently by Kemp (1964) and Vanek (1965) and proved by Ohyama (1972) and Kemp and Wan (1976). The validity of Kemp-Vanek-Ohyama-Wan theorem relies on the assumption the inter-country transfers under certain circumstances.

Ethiner (1998) finds that regionalism results from multilateralism, and then it will act as “an added guarantee for its survival”. Ethiner (1998) also shows that multilateral liberalization among rich countries may make rich-poor country FTAs more profitable. The conclusion of Ornelas (2005a) also provides a support for this idea that Free Trade Areas (FTAs) are good for the multilateral trading system, and can promote global free

trade. This line of research is in support of the claim that PTAs are building blocks.

On the other hand, there are also considerable amount of literature support the negative claim (stumbling block). Based on a pure exchange economy model, Kennan and Riezman (1990) indicates that due to increased market power, if the size of CU is large enough, it can improve the member countries' welfare over free trade, and becomes a barrier for that. By allowing countries deviate unilaterally, Bagwell and Staiger (1997) argue that the harmony between multilateral liberalization (as a global self-enforcing multilateral trade policy agreement) and CU is temporary.

Using intraindustry oligopolistic trade model, Saggi (2006) argues that PTAs are stumbling blocks for global free trade under symmetric country assumption. By clarifying the subtle differences of the effects under FTA and CU, he points out that the reason why FTA undermines global free trade is that it reduces nonmembers' incentives willingness to cooperate whereas a CU does so by reducing the incentives on members. Some other research focusing on FTA and multilateral liberalization generated similar negative messages¹.

To reconcile the diverging viewpoints in the literature, Yi (1996)² points out that the nature of the impact of a PTA, particularly a CU on global free trade depends on the regime of regionalism. Specifically, under "open regionalism" where a country can join the CU without consent from its members, global free trade can realize; while under "unanimous regionalism" where a nonmember country can entry only if when all the existing members in the CU admit it unanimously, CU will hinder global free trade.

¹Bond, Riezman and Syropoulos (2004) find that when external tariffs are constant, ROW's terms of trade may improve when FTA members adopt free internal trade hence incentive for outsiders to join an FTA will decrease. When ROW's tariffs are endogeneously determined, an FTA may prefer free internal trade only if its size is large enough.

Ornelas (2005b) shows that the trade creating effect of FTA, caused by the reduction of trade protections of member countries against nonmembers which facilitates the trade between them, can hamper the global free trade as it reduces the extra gains of nonmembers from multilateral liberalization.

²The discussion of Andriamananjara (2002) on Preferential Trading Arrangements has a similar result. It concludes that bloc expansion would result in global free trade if blocs have an open membership. However, that would not be the case if membership were selective: at some point, the members of the bloc would stop accepting new members and this would lead to the creation of a second bloc.

To our knowledge, the bulk of the existing literature study such problem under several-heterogeneous-country (usually three countries) or n -homogeneous-country setting. Such frameworks, though sometimes enable one to obtain close form solution, are in general not sufficient to capture multi-dimensional heterogeneity. Introducing asymmetries in the model is of practical importance since countries vary in technology, domestic competition, as well as market size. Yi (1996)'s result that in a symmetric world, under open regionalism, CU is a building block for global free trade, is not intuitively straightforward to be carried over to the asymmetric case.

This paper therefore aims to address the relationship between customs union and multilateral trade liberalization in an asymmetric world using coalition formation game approach. The model employed in the current paper is an n -heterogeneous-country Cournot model. Three dimensions of heterogeneity are considered: marginal cost of production, market size, and industry concentration. The main contributions of this paper include: (i) determining the endogenous formation path of the customs union; (ii) in contrast with Yi (1996), the current model concludes that if countries are myopic, under some circumstances, global free trade may not be achieved even under open regionalism in which a nonmember country can enter the CU without the consent of member countries, (iii) the negative result under myopic assumption is eliminated when countries become farsighted, that is, under farsightedness assumption, CU can expand until global free trade is achieved under open regionalism, even if countries presently suffer losses from participating the CU.

The rest of this paper is organized as follows: Section 2 introduces the basic model; Section 3 discusses the formation game of customs union under myopicism; Section 4 analyzes how farsightedness will improve the results in Section 3; and Section 5 offers some concluding remarks.

2 The Model

Traditionally, papers discussing the CU formation problem adopt the three-heterogeneous-country model or the n -homogeneous-country model. Such settings may have significant

drawbacks. In the former one, it usually takes the initial two-country CU as exogenously given, so models fail to characterize the complete formation process of a CU; for the later one, it does not take account of heterogeneity between countries, therefore the impact of heterogeneity on CU formation is left unanalyzed.

The n -heterogeneous-country model applied in this paper is set up to overcome such defects. Such setting enable us to have a better understanding on the impact of CU on multilateral liberalization both in terms of final outcome and the endogenous formation path of an self-enforcing CU.

In the process of CU formation, it is assumed that only one country is allowed to enter at a time (except the first stage where two countries who propose to initiate the CU). Throughout the paper, we assume incentive priority rule (IPR) where the country with the largest potential gain from participating the CU will become the first candidate member at each stage³. Therefore the priority (timing) of a country in joining the CU is indexed by the magnitude of its potential gains from joining the CU.

To make the results comparable, the regime of CU adopted in this model is “Open Regionalism” introduced in Yi (1996) in which a nonmember country can enter the CU without the consent of member countries. Such regime is also looser in the sense that internal stability condition does not apply in the myopic formation game.

For trackability, this paper assumes that the tariffs of nonmembers of the CU to all other countries and the tariff of the CU to the rest of the world are prohibitive, that is, all nonmembers are autarky and the CU does not have trade with rest of the world. A number of authors (Krugman (1991), Panagariya and Findlay (1994), Kannan and Riezman (1990)) argue that member countries have an incentive to raise their common external tariff. However, a recent report by World Bank (2000) finds little evidence in support of this result. Indeed, some of the existing works assume fixed external tariffs (Grossman and Helpman (1995), Krishna(1998)) which can be understood as the PTAs adherence to GATT’s Article XXIV. Though the fixed external tariff assumption is *ad hoc*, and eliminates “trade diversion” effect between a member and nonmember countries,

³The justification of IPR lies in the fact that for a benevolent domestic social planner, *ceteris paribus*, the higher the incentive (potential gains) for it to act is, the faster it takes action.

it has the merit to single out the tradeoff between trade creation and market displacement effects in the asymmetric country CU formation process.

We now introduce details of the model. There is one homogeneous good, and the aggregate utility in country i is assumed to have a quasi-linear form and the subsequent demand function of it in country i is $Q_i = \gamma_i(a - P_i)$, $i = 1 \dots n$, where Q_i and P_i are the total demand and price in country i , γ_i is a parameter characterizing country i 's market size, $\gamma_i \in \{1, \gamma\}$, $1 > \gamma > 0$. The number of profit-maximizing firms in country i is $m_i \in \{m_1, m_2\}$, $m_2 < m_1 < 2m_2$. Firms in the same country are homogeneous, i.e., they have the same marginal cost of production. In country i , the marginal cost is $c_i \in \{c_1, c_2\}$, $0 < c_2 < c_1$. Countries may differ in market size (γ_i), degree of concentration (m_i), and marginal cost of production (c_i), which are the three dimensions of heterogeneity considered in the model. Therefore there are eight kinds of countries, each of which is defined by a combination of the three heterogeneity parameters. For simplicity, we assume that the number of each kind of country is k ($k > 2$), so the total number of countries is $N = 8k$.

Table 1. All Kinds of Countries

Item \ No.	I	II	III	IV
Characteristics	$(1, m_1, c_1)$	(γ, m_1, c_1)	$(1, m_2, c_1)$	(γ, m_2, c_1)
Item \ No.	V	VI	VII	VIII
Characteristics	$(1, m_1, c_2)$	(γ, m_1, c_2)	$(1, m_2, c_2)$	(γ, m_2, c_2)

In following discussion, we define the social welfare of any nonmember country i as $W(\gamma_i, m_i, c_i)$. It is easy to see that the same kind of nonmembers have the same amount of social welfare, because they have the same heterogeneity parameters. When a nonmember country i enters the CU which has already had $l-1$ members, and becomes the l th member, its welfare will change from $W(\gamma_i, m_i, c_i)$ to $W(\gamma_i, m_i, c_i | l)$ which stands for the social welfare of member country i , when the size of the CU is l . Let $\Delta W(\gamma_i, m_i, c_i | l)$ stand for the welfare change when nonmember i become the l th member of the CU, then we have

$$\Delta W(\gamma_i, m_i, c_i | l) = W(\gamma_i, m_i, c_i | l) - W(\gamma_i, m_i, c_i).$$

This is the immediate welfare change for nonmember country i to become the l th member

of the CU. When global free trade is realized, let $G(\gamma_i, m_i, c_i)$ denote the welfare of country i .

Now, we begin our formal discussion of the Customs Union formation game.

3 Myopic Customs Union Formation Game

In this section, countries are assumed to be myopic, that is, when they make participation decisions, they only consider the immediate welfare change of participating the CU: if $\Delta W(\gamma_i, m_i, c_i | l) > 0$, country i enters the CU; otherwise, they refuse to join the union. Based on this assumption, this section has two main tasks, the first one is to find the endogeneous formation path of the self-enforcing CU, and the other one is to see whether the CU will continue its expansion until global free trade is realized.

3.1 Autarky

According to the assumption in Section 2, all countries have no trade with each others, namely autarky, before the CU is formed. In this case, the welfare of a country only consists of consumer surplus and domestic profits of firms. Let q_i denote the production of a single firm in country i in autarky. Under Cournot competition, we can get

$$q_i = \frac{\gamma_i (a - c_i)}{m_i + 1},$$

$$Q_i = m_i q_i = \frac{\gamma_i m_i (a - c_i)}{m_i + 1},$$

$$P_i = \frac{a + m_i c_i}{m_i + 1}.$$

Let π_i and CS_i stand for the profit of a single firm and consumer surplus respectively in country i under autarky, then we have

$$\pi_i = \frac{\gamma_i (a - c_i)^2}{(m_i + 1)^2},$$

$$CS_i = \frac{\gamma_i m_i^2 (a - c_i)^2}{2(m_i + 1)^2}.$$

Since under autarky the social welfare of country i is $W(\gamma_i, m_i, c_i) = m_i\pi_i + CS_i$, then from the results above, it is easy to obtain

$$W(\gamma_i, m_i, c_i) = \frac{\gamma_i m_i (a - c_i)^2}{(m_i + 1)^2} \left(1 + \frac{m_i}{2}\right).$$

This is the reserved utility of country i , and it is not affected by other factors outside the country. When the country is myopic, it will have incentive to join the CU, only if it could have a higher social welfare than this after becoming a CU member.

3.2 The First Two Members

A PTA may not reach its final size at a time, but expands until no countries would like to join it. EU and NAFTA are such examples. This paper wants to study how CU, a kind of PTA, expands under IPR. To carry out the study, the first step is to determine the first two members of the CU, because the beginning of the CU formation is that two autarky countries open their doors to have free trade with each other (these two still have no trade with others).

Assume that country i, j form the CU first, then country j (i) must be the one that brings i (j) the highest gain from forming the CU. When the two-country CU is formed, the welfare of member country i is

$$\begin{aligned} W(\gamma_i, m_i, c_i | 2) &= \underbrace{\frac{(\gamma_i + \gamma_j) m_i (a - (m_j + 1) c_i + m_j c_j)^2}{(m_i + m_j + 1)^2}}_{\text{the profits of firms in country } i} \\ &\quad + \underbrace{\frac{1}{2} \frac{\gamma_i ((m_i + m_j) a - m_i c_i - m_j c_j)^2}{(m_i + m_j + 1)^2}}_{\text{the consumer surplus of country } i}. \end{aligned} \quad (1)$$

The gain of country i from forming the CU with country j is

$$\begin{aligned} \Delta W(\gamma_i, m_i, c_i | 2) &= W(\gamma_i, m_i, c_i | 2) - W(\gamma_i, m_i, c_i) \\ &= \frac{(\gamma_i + \gamma_j) m_i (a - (m_j + 1) c_i + m_j c_j)^2}{(m_i + m_j + 1)^2} \\ &\quad + \frac{1}{2} \frac{\gamma_i ((m_i + m_j) a - m_i c_i - m_j c_j)^2}{(m_i + m_j + 1)^2} - \frac{\gamma_i m_i (a - c_i)^2}{(m_i + 1)^2} \left(1 + \frac{m_i}{2}\right) \end{aligned} \quad (2)$$

For country i , to determine its most appropriate trade partner is to choose a combination of γ_j, m_j, c_j that maximizes $\Delta W(\gamma_i, m_i, c_i | 2)$. Now, we differentiate $\Delta W(\gamma_i, m_i, c_i | 2)$ with respect to γ_j, m_j, c_j separately to see how it changes with these three heterogeneity parameters.

The derivative of $\Delta W(\gamma_i, m_i, c_i | 2)$ with respect to γ_j is

$$\frac{\partial \Delta W(\gamma_i, m_i, c_i | 2)}{\partial \gamma_j} = \frac{m_i (a - (m_j + 1)c_i + m_j c_j)^2}{(m_i + m_j + 1)^2}. \quad (3)$$

$m_i > 0, \frac{(a - (m_j + 1)c_i + m_j c_j)^2}{(m_i + m_j + 1)^2} > 0$, so $\frac{\partial \Delta W(\gamma_i, m_i, c_i | 2)}{\partial \gamma_j} > 0$, say, *ceteris paribus*, the welfare gain of country i from forming the CU is increasing with the market size of its trade partner. Therefore, country i will always choose to have free trade with a country owning a large market. In our model, $\gamma_j \in \{1, \gamma\}$, $1 > \gamma > 0$, so $\gamma_j = 1$.

This result is consistent with intuition. When country i and j have free access to each other's market, the social welfare of country i and j will both be affected in three aspects: (i) consumer surplus of a country will be increased due to increased competition; (ii) competition from foreign firms reduces domestic firms' profits in the home market; (iii) free access to foreign market brings profits to domestic firms. According to the three effects, given the degree of concentration and firms' marginal cost of production of country j unchanged, the first and second effects will be unaffected. At this time, when the market size of country j is increased, the foreign profits of firms in country i will be increased, and the welfare of country i after becoming a CU member will be higher. So country i always prefers to form a CU with a large-market country, *ceteris paribus*. In the rest of this subsection, we substitute γ_j with 1 to simplify our discussion.

To differentiate $\Delta W(\gamma_i, m_i, c_i | 2) |_{\gamma_j=1}$ with respect to m_j , we have

$$\frac{\partial \Delta W(\gamma_i, m_i, c_i | 2) |_{\gamma_j=1}}{\partial m_j} = \frac{(-m_i \gamma_i - 2m_i + \gamma_i m_j) a^2}{(m_i + m_j + 1)^3} + (*). \quad (4)$$

To simplify the discussion in the rest of this paper, we now assume that a is so large that the sign of a function of a is determined by the sign of the coefficient on the term with the highest order of a . Though this assumption is restrictive, it has its reasonability. In some industries producing primary goods, like oil and food, there are always non-zero demand, even though their prices are extremely high. Using a linear demand function like the one

in this paper to approximate their demand, there will inevitably be a very large a in the demand function.

Equation (4) is a quadratic function of a , where $(*)$ stands for the terms that do not include a^2 . According to the assumption above, to determine the sign of $\frac{\partial \Delta W(\gamma_i, m_i, c_i | 2) |_{\gamma_j=1}}{\partial m_j}$, we only need to examine the sign of $\frac{(-m_i \gamma_i - 2m_i + \gamma_i m_j)}{(m_i + m_j + 1)^3}$. Since $m_i, m_j \in \{m_1, m_2\}$, $m_2 < m_1 < 2m_2$, and $\gamma_i < 1$, we can prove that

$$\frac{(-m_i \gamma_i - 2m_i + \gamma_i m_j)}{(m_i + m_j + 1)^3} < \frac{(-m_i \gamma_i - 2m_i + \gamma_i 2m_i)}{(m_i + m_j + 1)^3} < 0. \quad (5)$$

Hence we have

$$\frac{\partial \Delta W(\gamma_i, m_i, c_i | 2) |_{\gamma_j=1}}{\partial m_j} < 0. \quad (6)$$

This implies that country i always prefers to form the CU with a country with a higher degree of concentration.

This result is intuitive. When country i and j can access each other's market freely, according to the three welfare effects, given the market size and firms' marginal cost of production in country j unchanged, a higher degree of concentration in country j will reduce the magnitude of increase in consumer surplus but raise the competitiveness of country i 's firms at home and abroad. In this model, the impact of a higher degree of concentration in country j on the later two effects are stronger than that on the first one, so country i prefers to form the CU with a country that has a higher degree of concentration. In the discussion of rest of this subsection, we substitute m_j with m_2 .

Now we look at how $\Delta W(\gamma_i, m_i, c_i | 2) |_{\gamma_j=1, m_j=m_2}$ changes with c_j ,

$$\frac{\partial \Delta W(\gamma_i, m_i, c_i | 2) |_{\gamma_j=1, m_j=m_2}}{\partial c_j} = \frac{m_2(3\gamma_i m_i - \gamma_i m_2)a}{(m_i + m_2 + 1)^2} + (*). \quad (7)$$

Since $m_i \in \{m_1, m_2\}$, $m_2 < m_1 < 2m_2$, we have $m_i < m_2$, $\frac{m_2(3\gamma_i m_i - \gamma_i m_2)a}{(m_i + m_2 + 1)^2} > 0$. $(*)$ stands for the terms that do not include a or its higher order, so the sign of (7) is determined by the term $\frac{m_2(3\gamma_i m_i - \gamma_i m_2)a}{(m_i + m_2 + 1)^2}$, therefore we have $\frac{\partial \Delta W(\gamma_i, m_i, c_i | 2) |_{\gamma_j=1, m_j=m_2}}{\partial c_j} > 0$. This result means that *ceteris paribus*, country i always prefers to form the CU with a country where firms' marginal cost of production is high.

The logic behind is that when country i and j have free access to each other's market, the three welfare effects listed above appear. Given the market size and degree of

concentration of country j unchanged, a higher marginal cost of production of country j will reduce the magnitude of increase in consumer surplus but raise the competitiveness of country i 's firms at home and abroad. In the model of this paper, the impact of a higher marginal cost of country j on the later two effects more than offsets that on the first one, so country i would like to have a free-trade partner with higher marginal cost of production.

According to the analysis above, in this model the most appropriate CU partner for any country i is a country with a larger market (1), higher degree of concentration (m_2) and higher marginal cost of production (c_1), therefore the first two countries of the CU will be the ones characterized by heterogeneity parameters 1, m_2 , c_1 , if such two countries really have incentive to form the CU. If such two do not have the incentive, then the CU will not be realized.

To test whether these two countries have incentive to form the CU, we need to see whether $\Delta W(1, m_2, c_1 | 2)$ is larger than zero.

$$\begin{aligned} \Delta W(1, m_2, c_1 | 2) &= W(1, m_2, c_1 | 2) - W(1, m_2, c_1) \\ &= \frac{1}{2} \frac{m_2 (a - c_1)^2 (2 + 3m_2)}{(2m_2 + 1)^2 (1 + m_2)^2}. \end{aligned} \quad (8)$$

Clearly, from equation (8) above, one observes that $\Delta W(1, m_2, c_1 | 2) > 0$, therefore the two countries characterized by 1, m_2 , c_1 have the incentive to form the CU, and they are the first two members of the CU according to IPR.

3.3 The Third Member

In the preceding subsection, we have determined the first two members of the CU. After the two-country CU is formed, other countries may have incentive to enter the union to have free trade with the first two members. This subsection is aimed to determine the third member country that participates the CU under IPR and open regionalism.

For any country i , its potential gain of becoming the third member is

$$\begin{aligned}
\Delta W (\gamma_i, m_i, c_i | 3) &= W (\gamma_i, m_i, c_i | 3) - W (\gamma_i, m_i, c_i) \\
&= \frac{(2 + \gamma_i) m_i (a - c_i + 2 m_2 c_1 - 2 m_2 c_i)^2}{(1 + m_i + 2 m_2)^2} \\
&\quad + \frac{1}{2} \frac{\gamma_i (m_i a - m_i c_i - 2 m_2 c_1 + 2 m_2 a)^2}{(1 + m_i + 2 m_2)^2} \\
&\quad - \frac{\gamma_i m_i (a - c_i)^2}{(m_i + 1)^2} \left(1 + \frac{m_i}{2}\right). \tag{9}
\end{aligned}$$

According to IPR assumed in this paper, to determine the third member of the CU, we need to find which one has the largest $\Delta W (\gamma_i, m_i, c_i | 3)$. Now, we analyse the derivatives of $\Delta W (\gamma_i, m_i, c_i | 3)$ with respect to γ_i, m_i, c_i separately to find the combination of the three heterogeneity parameters that maximizes $\Delta W (\gamma_i, m_i, c_i | 3)$.

The derivative of $\Delta W (\gamma_i, m_i, c_i | 3)$ with respect to γ_i is

$$\frac{\partial \Delta W (\gamma_i, m_i, c_i | 3)}{\partial \gamma_i} = \frac{2 m_2 (m_2 - m_i - m_i^2) a^2}{(1 + m_i + 2 m_2)^2 (1 + m_i)^2} + (*). \tag{10}$$

(*) stands for the terms that do not include a^2 , so the sign of (10) is determined by $\frac{2 m_2 (m_2 - m_i - m_i^2) a^2}{(1 + m_i + 2 m_2)^2 (1 + m_i)^2}$. Since $m_i \in \{m_1, m_2\}$, $m_2 < m_1 < 2 m_2$, we have $\frac{\partial \Delta W (\gamma_i, m_i, c_i | 3)}{\partial \gamma_i} < 0$. This means that *ceteris paribus*, countries with smaller markets always enjoy more welfare gain from participating the CU than do the ones with larger markets.

This result is understandable. When country i has free trade with the first two members of the CU, its social welfare will be affected in three aspects as listed in the subsection above: consumer surplus, profits of domestic firms in home market and profits of domestic firms in foreign markets. Assuming that the degree of concentration and firms' marginal cost of production of country i are given, the increase in consumer surplus and decrease in profits of domestic firms in home market induced by free trade will change with the market size of country i . In the model of this paper, the impact of market size change on the decrease in profits of domestic firms in home market is stronger than that on the increase in consumer surplus, hence countries with smaller markets enjoy more welfare gain from participating the CU. To simplify our later discussion, we substitute γ_i with γ .

To differentiate $\Delta W(\gamma, m_i, c_i | 3)$ with respect to m_i , we have

$$\frac{\partial \Delta W(\gamma, m_i, c_i | 3)}{\partial m_i} = \frac{2 \left[\begin{array}{l} m_i^3 (2m_2 + \gamma m_2 - m_i - 2) \\ + (\gamma m_i^3 m_2 + 3\gamma m_i^2 m_2 - 4\gamma m_2^3) \\ + (6m_i^2 m_2 - 6\gamma m_2^2 m_i) + (6m_i m_2 - 6\gamma m_2^2) \\ + (2m_2 - \gamma m_2) + 2m_i + 1 \end{array} \right] a^2}{(1 + m_i + 2m_2)^3 (1 + m_i)^3} + (*). \quad (11)$$

where $(*)$ stands for the terms that do not include a^2 , so the sign of (11) is determined by the terms in the brackets. We have assumed that $m_i \in \{m_1, m_2\}$, $m_2 < m_1 < 2m_2$, $\gamma < 1$, so it is easy to verify that the sum of terms in each parentheses is positive, then we have $\frac{\partial \Delta W(\gamma, m_i, c_i | 3)}{\partial m_i} > 0$. This result indicates that countries with more firms will have a higher incentive to become the third CU member.

The interpretation of the result is that when country i has free trade with the first two members of the CU, according to the three welfare effects, given the market size and firms' marginal cost of production in country i unchanged, a higher degree of concentration in country i will reduce the magnitude of increase in consumer surplus but raise the competitiveness of country i 's firms at home and abroad. In this model, the impact of a higher degree of concentration in country i on the later two effects are stronger than that on the first one, so countries with higher degree of concentration enjoy more welfare gain from participating the CU. In following discussion, we use m_1 to substitute m_i .

The derivative of $\Delta W(\gamma, m_1, c_i | 3)$ with respect to c_i is

$$\frac{\partial \Delta W(\gamma, m_1, c_i | 3)}{\partial c_i} = \frac{2m_1 \left[\begin{array}{l} (\gamma m_2 - 4m_1) + (4\gamma m_2^2 - 8m_2 m_1) \\ + (2\gamma m_1 m_2^2 - 4m_2 m_1^2) - 2 \\ - \gamma m_2 m_1^2 - 4m_2 - 2m_1^2 \end{array} \right] a}{(1 + 2m_2 + m_1)^2 (m_1 + 1)^2} + (*). \quad (12)$$

The sign of (12) is determined by the terms in the brackets, as $(*)$ stands for the terms that do not include a or higher order of a . Since $m_i \in \{m_1, m_2\}$, $m_2 < m_1 < 2m_2$, $\gamma < 1$, it is easy to verify that the sum of terms in each parentheses is positive, then we have $\frac{\partial \Delta W(\gamma, m_1, c_i | 3)}{\partial c_i} < 0$. This result indicates that countries with higher production efficiency

always enjoy more welfare gain from participating the CU than do the ones with lower efficiency.

This result is easy to understand. When country i has free trade with the first two members of the CU, according to the three welfare effects, given the market size and degree of concentration in country i unchanged, a lower firms' marginal cost of production in country i will The paper assumes that the priority (timing) of a country in joining the CU is indexed by the magnitude of its potential gains from joining the CU. In this model, the impact of a lower firms' marginal cost of production in country i on the later two effects are stronger than that on the first one, so countries with higher production efficiency enjoy more welfare gain from joining the CU.

Now, under IPR, we have found that a country characterized by heterogeneity parameters 1, m_1 , c_2 are most likely to become the third member of the CU. However, to determine whether such a country will join the CU, we need to examine the sign of $\Delta W(\gamma, m_1, c_2 | 3)$. The expression of $\Delta W(\gamma, m_1, c_2 | 3)$ is

$$\Delta W(\gamma, m_1, c_2 | 3) = \frac{2[\gamma m_2^2 + m_1 + (2m_1^2 - \gamma m_1 m_2) + (m_1^3 - \gamma m_2 m_1^2)] a^2}{(1 + 2m_2 + m_1)^2 (m_1 + 1)^2} + (*). \quad (13)$$

(*) stands for the terms that do not include a^2 , so the sign of (13) is determined by the coefficient on a^2 . The sum of terms in each parentheses is positive, so $\Delta W(\gamma, m_1, c_2 | 3) > 0$, that is, a country characterized by γ, m_1, c_2 will become the third CU member under open regionalism.

3.4 Incentive for the Last Kind

Similar to the analysis of determining the third CU member, we can demonstrate that the fourth member is also a country with heterogeneity parameters γ, m_1, c_2 (see Appendix A). Moreover, using mathematical induction, we prove that the members of the CU from the third to the $(k + 2)$ th are all the countries characterized by γ, m_1, c_2 (see Appendix B).

Now, let us see how the CU will expand after all γ, m_1, c_2 countries have joined it. We have known that when any country i joins the CU, its social welfare will be affected

in three aspects: consumer surplus, profits of domestic firms in home market and profits of domestic firms in foreign markets. According to the results and relevant analysis in Subsection 1 and 2 of this section, we can find that the three heterogeneity parameters γ_i, m_i, c_i affect the change of social welfare induced by free trade in the three aspects as follows,

Table 2. How the parameters affect the change of social welfare in the three aspects of country i

\setminus	Increase in CS	Decrease in PHM	Increase in PFM	Change of SW
γ_i	+	+	0	-
m_i	-	-	+	+
c_i	+	+	-	-

Note: CS—Consumer Surplus; PHM—Profits in Home Market; PFM—Profits in Foreign Market; SW—Social Welfare

+ : Positively Correlated, *ceteris paribus*

- : Negatively Correlated, *ceteris paribus*

0 : Noncorrelated, *ceteris paribus*

From the last column, we can see that the change of social welfare of country i induced by free trade decreases with γ_i, c_i , but increases with m_i . That is why countries characterized by γ, m_1, c_2 always have more incentive to participate the CU than others under IPR. The results in the last column imply the expansion path of the CU:

$$(\gamma, m_1, c_2) \longrightarrow \left\{ \begin{array}{l} \left\{ \begin{array}{l} (\gamma, m_1, c_1) \longrightarrow (\gamma, m_2, c_1) \\ (\gamma, m_2, c_2) \end{array} \right. \\ \dots\dots\dots \\ (1, m_1, c_2) \longrightarrow \left\{ \begin{array}{l} (1, m_1, c_1) \\ (1, m_2, c_2) \end{array} \right. \end{array} \right. \longrightarrow (1, m_2, c_1).$$

This graph means that countries with γ, m_1, c_2 always have the most incentive to enter the CU, countries with γ, m_1, c_1 and γ, m_2, c_2 have more incentive than countries with γ, m_2, c_1 to become the member of the CU and countries with $1, m_1, c_2$, if they could benefit from becoming the CU members, will become members of the CU earlier than countries with $1, m_1, c_1$ and $1, m_2, c_2$ (to determine the exact expansion path in the braces, we need further assumptions for our model). $1, m_2, c_1$ countries are always the ones least likely to become the CU members (except the first two CU members).

Now, it is time to answer the interesting open question: whether a CU is building or stumbling block for the realization of global free trade? In this model, to find an answer, we just need to test whether countries characterized by $1, m_2, c_1$ would like to participate the CU, assuming all other countries have incentive to participate the CU. If all the $1, m_2, c_1$ countries remained (the first two members of the CU are such countries) could benefit from participation, then global free trade will be realized; if not, global free trade will be hindered by the CU.

The welfare change of a $1, m_2, c_1$ country after participating the CU, just after all other countries have become members of the CU, is

$$\Delta W(1, m_2, c_1 | 7k + 3) = \frac{\left[\begin{array}{c} (16 m_1^2 + 9 m_2^2 + 24 m_1 m_2) k^2 \\ + \left(\begin{array}{c} -8 m_1 m_2^2 + 8 \gamma m_2 + 6 m_2 \\ + 8 m_1 m_2 + 16 \gamma m_2^2 \\ + 18 m_2^2 + 8 \gamma m_2^3 \\ + 8 m_2^2 + 4 m_2 \end{array} \right) k \end{array} \right] a^2}{2(4 k m_1 + 3 m_2 + 3 k m_2 + 1)^2 (m_2 + 1)^2} + (*). \quad (14)$$

(*) stands for the terms that do not include a^2 , so the sign of (14) is determined by the terms in the brackets. Though most of the terms in the brackets are positive, it is still possible that the coefficient on a^2 is negative.

Proposition 3.1 *There is a subset of $\{(k, \gamma, m_1, m_2) | k > 2, 0 < \gamma < 1, m_2 < m_1 < 2m_2\}$ which can satisfy $\Delta W(1, m_2, c_1 | 7k + 3) < 0$, that is, the countries characterized by $1, m_2, c_1$ do not have incentive to participate the CU after all other countries have become the CU members under open regionalism.*

Proof. This is proved by contradiction. Suppose the proposition is not true. We can construct a counter-example below:

1. when $\gamma = 0.1, m_1 = 1500, m_2 = 1000$, the numerator of the coefficient on a^2 is $-1.116839320 \times 10^{10} \cdot k + 81000000 \cdot k^2 + 8004000$. So when $k \in \{x | 3 \leq x \leq 137, x \in Z\}$, $\Delta W(1, m_2, c_1 | 7k + 3) < 0$;

2. when $\gamma = 0.01$, $m_1 = 1500$, $m_2 = 1000$, the numerator of the coefficient on a^2 is $-1.188983392 \times 10^{10} \cdot k + 81000000 \cdot k^2 + 8004000$. So when $k \in \{x \mid 3 \leq x \leq 146, x \in Z\}$, $\Delta W(1, m_2, c_1 \mid 7k + 3) < 0$.

■

Different from the result of Yi (1996), this proposition indicates that even under open regionalism a CU may be a stumbling block for global free trade. This result is due to the heterogeneities introduced in this paper.

4 Farsightedly Stable Customs Union

In the last subsection of the preceding section, we have demonstrated that when countries make participation decisions according to their immediate welfare change after joining the CU, global free trade could not be realized under some conditions. Will the results above be changed when countries are farsighted? This section is designed to answer this question.

Since the countries left outside the CU are all the ones with $1, m_2, c_1$, we assume that their order of making participation decisions is randomly determined. “Farsighted” in this paper means that when a former country makes participation decision, it takes account of later countries’ decisions.

Now, let us first solve out the condition for the last country to participate the CU after all other contries have become members of the CU. The welfare change of the last $1, m_2, c_1$ country after participating the CU is

$$\Delta W(1, m_2, c_1 \mid 8k) = \frac{\left[\begin{array}{l} (16 m_2^2 + 32 m_2 m_1 + 16 m_1^2) k^2 \\ + \left(\begin{array}{l} -8 m_2^2 m_1 - 16 m_2 m_1 \\ + 8 m_2 \gamma + 8 m_2^3 \gamma \\ + 16 m_2^2 \gamma + 8 m_2 \\ - m_2^2 - 2 m_2 \end{array} \right) k \end{array} \right] a^2}{2(4 k m_1 + 1 + 4 k m_2)^2 (m_2 + 1)^2} + (*). \quad (15)$$

Understandably, $\Delta W(1, m_2, c_1 \mid 8k) = G(1, m_2, c_1) - W(1, m_2, c_1)$, where $G(1, m_2, c_1)$ means the welfare of a $1, m_2, c_1$ country under global free trade and $G(1, m_2, c_1) - W(1, m_2, c_1)$

denotes the welfare gain (or loss) of a $1, m_2, c_1$ country from the realization of global free trade. (*) stands for the terms that do not include a^2 , so the sign of (15) is determined by the terms in the brackets.

When $(16 m_2^2 + 32 m_2 m_1 + 16 m_1^2) k^2 + \begin{pmatrix} -8 m_2^2 m_1 - 16 m_2 m_1 + 8 m_2 \gamma \\ +8 m_2^3 \gamma + 16 m_2^2 \gamma + 8 m_2 \end{pmatrix} k - m_2^2 - 2 m_2 > 0$, we have $\Delta W(1, m_2, c_1 | 8k) > 0$, namely the last country has incentive to become the CU member and a $1, m_2, c_1$ country can gain from the realization of global free trade.

Proposition 4.1 *There is a subset of $\{(k, \gamma, m_1, m_2) | k > 2, 0 < \gamma < 1, m_2 < m_1 < 2m_2\}$ which can satisfy $\Delta W(1, m_2, c_1 | 7k + 3) < 0$ and $\Delta W(1, m_2, c_1 | 8k) > 0$.*

Proof. To simplify our discussion, we let M_1 and M_2 stand for the coefficients of $\Delta W(1, m_2, c_1 | 7k + 3)$ and $\Delta W(1, m_2, c_1 | 8k)$ on a^2 respectively, then we have $M_2 - M_1 = \frac{m_2(k-3)(8km_1+7km_2+3m_2+2)(8km_2\gamma-8km_1-1)}{2(4km_1+1+3m_2+3km_2)^2(4km_1+1+4km_2)^2}$. So when $k > 3$, $M_2 - M_1 > 0$. This implies that there is possibility of finding such a subset satisfying $\Delta W(1, m_2, c_1 | 7k + 3) < 0$ and $\Delta W(1, m_2, c_1 | 8k) > 0$.

It is difficult to work out the whole subset, but we can give some points of the subset to demonstrate its existence as in Proposition 3.1.

1. when $\gamma = 0.9$, $m_1 = 1050$, $m_2 = 1000$, the numerators of M_1 and M_2 are respectively $51840000k^2 - 1.159186800 \times 10^9 k + 8004000$ and $67240000k^2 - 1.202384800 \times 10^9 k - 1002000$. So when $k \in \{x | 18 \leq x \leq 22, x \in Z\}$, we have $\Delta W(1, m_2, c_1 | 7k + 3) < 0$ and $\Delta W(1, m_2, c_1 | 8k) > 0$.
2. when $\gamma = 0.9$, $m_1 = 1010$, $m_2 = 1000$, the numerators of M_1 and M_2 are respectively $49561600k^2 - 8.39506800 \times 10^8 k + 8004000$ and $64641600k^2 - 8.817448000 \times 10^8 k - 1002000$. So when $k \in \{x | 14 \leq x \leq 16, x \in Z\}$, we have $\Delta W(1, m_2, c_1 | 7k + 3) < 0$ and $\Delta W(1, m_2, c_1 | 8k) > 0$.

■

According to Proposition 4.1, we can realize that under a specific set of (k, γ, m_1, m_2) , though $1, m_2, c_1$ countries will suffer immediate welfare loss from joining the CU, they

could gain from the realization of global free trade. At this time, if these countries change from being myopic to being farsighted, global free trade can be realized even under the CU.

The intuition is of the followings: :

- When the 1st to the $(k - 3)$ th $1, m_2, c_1$ countries left outside the CU under myopicism join the CU, the last one will definitely follow them to become a CU member;
 - Under the fact above, when 1st to the $(k - 4)$ th $1, m_2, c_1$ countries left outside the CU under myopicism join the CU, the $(k - 3)$ th one will follow them to become a CU member, as it knows that the last one will follow to become a CU member;
 - Under the fact above, when 1st to the $(k - 5)$ th $1, m_2, c_1$ countries left outside the CU under myopicism join the CU, the $(k - 4)$ th one will follow them to become a CU member, as it knows that the $(k - 3)$ th one and the last one will follow to become CU members;
- ⋮
- Under the fact above, the 1st $1, m_2, c_1$ countries left outside the CU under myopicism will join the CU, as it knows that the 2nd one to the last one will follow to become CU members.

Therefore countries left outside the CU will all participate it under farsighted rationality assumption.

Now, we can claim that farsightedness could improve the result under myopic assumption and in the specific set of (k, γ, m_1, m_2) referred in Proposition 4.1.

5 Concluding Remarks

This paper adopts an n -heterogeneous-country model to analyse whether a CU is a building block or stumbling stone for the realization of global free trade. Though this interesting question has been widely studied, to the best of our knowledge, this paper is the first one to address the path of the CU expansion under asymmetric settings.

We defined incentive priority rule in this paper. Under this rule, we find the expansion path of the CU: the first two members of the CU are countries with the larger market, less firms and lower production efficiency (what is interesting is that such countries are also the ones having the least incentive to become members of the CU!); the third member to the $(k + 2)$ th one are all the countries with the smaller market, higher degree of concentration and lower marginal cost of production.

When assuming that countries are myopic, the current paper finds that even under open regionalism, global free trade may not be realized under some conditions. This result is due to the three dimensions of heterogeneity introduced in the model. Some previous literature (Yi (1996) and Andriamananjara (2002)) which does not take such heterogeneity parameters draws opposite conclusion.

Another finding of this paper is that under some conditions, farsightedness could improve the result obtained under myopic assumption. Specifically, when parameters in this model satisfy some conditions, farsighted countries will bear transitory welfare loss and choose to join the CU till global free trade is realized.

The objective of this paper is to offer a complete understanding of the relationship between CU and global free trade. It helps to learn which types of countries have least incentive to cooperate in CU expansion. In terms of policy recommendations, such countries should be given special treatment or priority in order to achieve global free trade via CU expansion.

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6 Appendix A

In this appendix, we will demonstrate the fourth member of the CU is a country characterized by γ, m_1, c_2 .

For any country i , its welfare change after becoming the fourth member is

$$\begin{aligned} \Delta W(\gamma_i, m_i, c_i | 4) &= \frac{(2 + \gamma + \gamma_i) m_i (a - c_i + 2 m_2 (c_1 - c_i) + m_1 (c_2 - c_i))^2}{(m_1 + 1 + m_i + 2 m_2)^2} \\ &+ \frac{1}{2} \frac{\gamma_i ((m_i + 2 m_2 + m_1) a - c_i m_i - 2 c_1 m_2 - m_1 c_2)^2}{(m_1 + 1 + m_i + 2 m_2)^2} \\ &- \frac{\gamma_i m_i (a - c_i)^2 (1 + \frac{m_i}{2})}{(m_i + 1)^2}. \end{aligned}$$

To differentiate $\Delta W(\gamma_i, m_i, c_i | 4)$ with respect to γ_i , we can get

$$\frac{\partial \Delta W(\gamma_i, m_i, c_i | 4)}{\partial \gamma_i} = \frac{(2 m_2 + m_1 - 2 m_i^2 - 2 m_i) (m_1 + 2 m_2) a^2}{2 (m_1 + 1 + m_i + 2 m_2)^2 (m_i + 1)^2} + (*).$$

Since $m_i \in \{m_1, m_2\}$, $m_2 < m_1 < 2 m_2$, we have $\frac{\partial \Delta W(\gamma_i, m_i, c_i | 4)}{\partial \gamma_i} < 0$. Therefore, countries with small markets have more incentive to participate the CU.

To differentiate $\Delta W(\gamma, m_i, c_i | 4)$ with respect to m_i , we can obtain

$$\begin{aligned} \frac{\partial \Delta W(\gamma, m_i, c_i | 4)}{\partial m_i} &= \frac{\left[\begin{aligned} &(3 \gamma m_1 m_i^3 - \gamma m_i^4) \\ &+ (6 \gamma m_1 m_i^2 - 3 \gamma m_1^2 m_i) \\ &+ \left(\begin{aligned} &6 \gamma m_2 m_i^3 - 12 \gamma m_2 m_1 m_i \\ &- 12 \gamma m_1 m_2^2 - 12 \gamma m_2^2 m_i \\ &- 8 \gamma m_2^3 \end{aligned} \right) \\ &+ (12 m_2 m_i - 12 \gamma m_2^2) + (6 m_1 m_i^2 - \gamma m_1^3) \\ &+ (6 m_1 m_i - 3 \gamma m_1^2) + (4 m_2 m_i^3 - 2 m_i^4) \\ &+ (12 m_2 m_i^2 - 2 \gamma m_i^3 - 4 m_i^3) \\ &+ (2 m_1 m_i^3 - 12 \gamma m_2 m_1 - 6 \gamma m_1^2 m_2) \\ &+ 12 \gamma m_2 m_i^2 + 4 m_i + 4 m_2 + 3 \gamma m_1 m_i \\ &+ 2 + \gamma + 6 \gamma m_2 m_i + 2 \gamma m_i + 2 m_1 \end{aligned} \right] a^2}{(m_1 + m_i + 2 m_2 + 1)^3 (m_i + 1)^3} + (*). \end{aligned}$$

The sum of terms in each pair of parenthesis is positive, so the coefficient on a^2 are positive, then $\frac{\partial \Delta W(\gamma, m_i, c_i | 4)}{\partial m_i} > 0$. This result indicates that countries with more firms will have more incentive to become the fourth CU member.

To differentiate $\Delta W(\gamma, m_1, c_i | 4)$ with respect to c_i , we get

$$\frac{\partial \Delta W(\gamma, m_1, c_i | 4)}{\partial c_i} = - \frac{\begin{bmatrix} 4 + 2\gamma + 5\gamma m_1 + 2\gamma m_2 \\ +12m_1 + 8m_2 + 8m_2m_1^2 \\ +2\gamma m_1^2m_2 + 16m_2m_1 + 4\gamma m_1^2 \\ +2\gamma m_1^3 + (12m_1^2 - 8\gamma m_2^2) \\ + (4m_1^3 - 4\gamma m_1m_2^2) \end{bmatrix} a}{(2m_1+2m_2+1)^2(m_1+1)^2} + (*).$$

The sum of terms in each pair of parenthesis is positive, so $\frac{\partial \Delta W(\gamma, m_1, c_i | 4)}{\partial c_i} < 0$, which means that countries with higher efficiency will have more incentive to become the fourth CU member.

According to the analysis above, the fourth member of the CU under IPR is a country with γ, m_1, c_2 .

7 Appendix B

In this appendix, we will use mathematical induction to prove that the members of the CU from the third to the $(k+2)$ th are all the countries characterized by γ, m_1, c_2 .

1. In Section 2, we have demonstrated that the third member of the CU under IPR is a γ, m_1, c_2 country;
2. We assume that the $(n+2)$ th member of the CU is a γ, m_1, c_2 country;
3. Now, we prove that the $(n+3)$ th member is also a γ, m_1, c_2 country:

For any country i , its welfare change after becoming the $(n+3)$ th member is

$$\begin{aligned} \Delta W(\gamma_i, m_i, c_i | n+3) &= \frac{(2+n\gamma+\gamma_i)m_i(a-c_i+2m_2(c_1-c_i)+nm_1(c_2-c_i))^2}{(nm_1+1+m_i+2m_2)^2} \\ &+ \frac{\gamma_i((m_i+2m_2+nm_1)a-c_im_i-2c_1m_2-nm_1c_2)^2}{2(nm_1+1+m_i+2m_2)^2} \\ &- \frac{\gamma_im_i(a-c_i)^2(1+\frac{m_i}{2})}{(m_i+1)^2}. \end{aligned}$$

To differentiate $\Delta W(\gamma_i, m_i, c_i | n+3)$ with respect to γ_i , we get

$$\frac{\partial \Delta W(\gamma_i, m_i, c_i | n+3)}{\partial \gamma_i} = \frac{(2m_2 - 2m_i - 2m_i^2 + nm_1)(nm_1 + 2m_2)a^2}{2(nm_1 + 1 + m_i + 2m_2)^2(m_i + 1)^2} + (*).$$

k is the number of a specific kind of country, so practically, k is a two-digit number.

Since $n < k$, and m_1, m_2 are at least four-digit numbers, we have $\frac{\partial \Delta W(\gamma_i, m_i, c_i | n+3)}{\partial \gamma_i} < 0$.

Therefore, countries with small markets have more incentive to participate the CU.

To differentiate $\Delta W(\gamma, m_i, c_i | n+3)$ with respect to m_i , we could obtain

$$\frac{\partial \Delta W(\gamma, m_i, c_i | n+3)}{\partial m_i} = - \left\{ \begin{array}{l} \left[\begin{array}{l} -3\gamma m_1 m_i - \gamma m_1 \\ + (3\gamma m_1^2 - 3\gamma m_1 m_i^2) \\ + \left(\begin{array}{l} 3\gamma m_1^2 m + \gamma n m_1^3 \\ + 6\gamma m_1^2 m_2 - \gamma m_1 m_i^3 \end{array} \right) \end{array} \right] n^2 \\ + \left[\begin{array}{l} -2\gamma m_2 - \gamma - 6\gamma m_2 m_i^2 - 2m_1 \\ -6m_1 m_i - 6\gamma m_2 m_i - 2m_1 m_i^3 \\ + (2\gamma m_i^3 - 6m_1 m_i^2) + (\gamma m_1 - 2\gamma m_i) \\ + (12\gamma m_2 m_1 - 3\gamma m_1 m_i^2) \\ + (12\gamma m_2 m_1 m_i + 12\gamma m_1 m_2^2 - 2\gamma m_1 m_i^3) \\ + (\gamma m_i^4 - 2\gamma m_2 m_i^3) \end{array} \right] n \\ + \left[\begin{array}{l} -2 - 6\gamma m_2 m_i^2 - 4m_i \\ + (2m_i^4 - 4m_2 m_i^3) + (2\gamma m_2 - 4m_2) \\ + (12\gamma m_2^2 - 12m_2 m_i) + (4m_i^3 - 12m_2 m_i^2) \\ + (8\gamma m_2^3 + 12\gamma m_2^2 m_i - 4\gamma m_2 m_i^3) \end{array} \right] \end{array} \right\} a^2 + (*).$$

The sum of terms in each pair of parenthesis is negative, so the sum of terms in each brackets is negative, then we have $\frac{\partial \Delta W(\gamma, m_i, c_i | n+3)}{\partial m_i} > 0$. This result indicates that countries with more firms will have more incentive to become the $(n+3)$ th CU member.

The derivative of $\Delta W(\gamma, m_1, c_i | n+3)$ with respect to c_i is

$$\frac{\partial \Delta W(\gamma, m_1, c_i | n+3)}{\partial c_i} = - \frac{m_1 \left[\begin{array}{l} 4 + 2n\gamma m_1^2 + n^2\gamma m_1^3 \\ + 2n^2\gamma m_1 + 4m_1^2 + \gamma n m_1^3 \\ + 3\gamma n m_1 + 4n m_1 + 4n\gamma m_2 \\ + 2\gamma m_1^2 m_2 + 16m_2 m_1 \\ + 4n m_1^3 + 2n\gamma + 2\gamma n^2 m_1^2 \\ + (8m_2 m_1^2 - 4\gamma m_1 m_2^2) \\ + (8n m_1^2 - 8\gamma m_2^2) \\ + (8m_1 - 2\gamma m_2) + 8m_2 \end{array} \right] a}{(nm_1 + m_i + 2m_2 + 1)^2 (m_i + 1)^2} + (*).$$

Since the sum of terms in each pair of parenthesis is positive, so $\frac{\partial \Delta W(\gamma, m_1, c_i | n+3)}{\partial c_i} < 0$,

which means that countries with higher efficiency will have more incentive to become the $(n+3)$ th CU member.

Therefore according to the analysis above, the $(n+3)$ th CU member is also a γ, m_1, c_2 country.

Till now, we have finished the proof that the members of the CU from the third to the $(k+2)$ th are all the countries characterized by γ, m_1, c_2 .