Financial Frictions and International Trade

Ruanjai Suwantaradon*

School of Economics, Singapore Management University

August 18, 2008

Abstract

This paper studies the effects of financial market imperfections on a firm's operating and exporting decisions. I introduce financial frictions into a trade model with heterogeneous firms along the line of Melitz (2003). With the presence of financial constraints, even among a group of firms with the same productivity level, firms that are more financially constrained operate on a less efficient scale, and as a result, may no longer find operating and/or exporting profitable. In addition, financial frictions may create a distortion compared to the Melitz (2003) world since operation and export participation may be undertaken by those with better access to finance than by those with higher productivity. Furthermore, financial frictions can have persistent effects on firms' dynamics. Productive firms with very low starting net worth will never accumulate enough to overcome credit constraints and, therefore, will never start operating and, subsequently, never export even if they are very productive. Using data from the World Bank Enterprise Surveys for Brazil and Chile, I find evidence that supports the model's prediction that exporters are likely those that are less financially constrained even after controlling for productivity and sectoral effects.

JEL Classifications: F10, F12, F14, F36, G20, G32. Keywords: financial frictions, borrowing constraints, firm heterogeneity, export participation

*I would like to thank Tim Kehoe, Cristina Arellano, Pedro Amaral, Erzo Luttmer, and Fabrizio Perri for valuable suggestions and comments. I also thank participants in the Workshop in Trade and Development at the University of Minnesota, Midwest International Economics Meetings, Minneapolis, Minnesota, April 2007, and Bag Lunch Seminar, Federal Reserve Bank of Minneapolis, Minneapolis, Minnesota, December 14 2007. Tel.: +65-9106-6710; e-mail: ruanjais@smu.edu.sg

1 Introduction

Does finance matter for export participation? Although the importance of financing constraints has been well documented on the determinants of firm dynamics,[1] it has been shown only recently, and with much more limited data, that a similar conclusion holds for firm export participation decisions.[2] This may be due to the fact that international trading imposes additional costs on firms. As has been documented by much empirical literature, besides costs due to customs and trade regulations, firms also face other fixed and sunk costs such as the costs of acquiring information about the market conditions abroad or establishing distribution channels.[3] Thus, natuarally firms' ability to finance these costs may play an important role in the firms' export decisions and that more developed financial markets will be able to better provide firms with financing, reducing the reliance of firms on internal funds to pay the costs involved in international trade.[4]

In this paper, I investigate one possible channel through which financial frictions affect firms' export participation decisions using a model of international trade with heterogeneous firms along the line of Melitz (2003).[5] However, departing from the norm of this type of international trade models with labor as the only input of production, I allow for capital as the only input, since with this setup, the analysis of the effects of financial frictions on firms' export decisions and dynamics is much simpler. In particular, the model features monopolistically competitive firms facing credit market frictions in raising capital necessary for production and financing fixed costs of exporting. Thus, the financial frictions directly affect the firms' level of production and dynamics by limiting the size of the projects that firms can finance and the amount of net worth they can accumulate. Therefore, in this model, firms are heterogeneous not only in the productivity level, as in the standard model, but also in the amount of net worth they have accumulated. Assuming that the amount of debt a firm can obtain is directly proportional to its net worth, the firm's output level and the decision whether to export will, therefore, not only depend on its productivity level, but also on its net worth.

The model predicts that credit constraints influence firms' selection into export markets: in order for a firm to profit from exporting, it must not only have high enough productivity but also a high enough level of net worth accumulation. Firms with low productivity and a low level of net worth will be more profitable by serving only the domestic market. Firms with higher net worth will find exporting more attractive. In addition, due to the general equilibrium effect, credit market frictions may create a distortionary effect: namely, export participation is undertaken by those firms with better access to finance than by those with higher productivity. Using the World Bank Enterprise Surveys for Brazil and Chile, I provide evidence complementary to the earlier literature and supporting the model's predictions that the firms' financial position is positively related to the propensity to export and exporting firms are less financially constrained.

[1] Cooley and Quadrini (2001), for example, introduce financial frictions into a model of industry dynamics and show that the model can account for the simultaneous dependence of firm dynamics on size, age, and financial characteristics of the firms.

[2] Greenaway et al. (2007) find that U.K. firms that are less financially constrained are more likely to export, and balance sheet variables are important determinants of firms' exporting decisions. Campa and Shaver (2002) find that liquidity constraints are less binding for exporters than for non-exporters for Spanish firms. Zia (2005) measures the effects of export subsidies on firms in Pakistan and identifies the privately owned firms as being financially constrained, and, thus, suggests that export credit subsidies should primarily be targeted to small, privately owned firms.

[3] Roberts and Tybout (1999) and Bernard and Jensen (2001), for example, have emphasized the sunk costs of exporting.

[4] Manova (2008b) finds that firms face credit constraints in the financing of both fixed and variable export costs.

[5] Melitz (2003) highlights the role of firm productivity in explaining a firm's participation in exporting.

The model also features the property that financial frictions can have persistent effects on firms' entry decisions.[6] In particular, productive firms with very low starting net worth may not choose to save to overcome credit constraints and, therefore, will never start operating and/or exporting even if they are very productive. This differs from the usual view that firms can always save their way out of the borrowing constraints. Thus, this result seems to support the usual arguments justifying policies in favor of small enterprises since, in the presence of financial market imperfections, those that are viable in the long run may fail to enter as they do not have funds to cover the costs in the early years.

This paper contributes to the growing literature on financial constraints and export participation. Prior researchers, however, have only augmented the Melitz (2003) model with an additional dimension of firm heterogeneity that is exogenously given. Chaney (2005), for example, focuses on liquidity-constrained exporting firms, assuming that firms inherit an exogenous amount of liquidity while Manova (2008b) assumes that firms can only partially finance fixed cost of trade internally with this fraction differs exogenously across firms. Garcia-Vega and Guariglia (2007) analyze the effects of firms' exogenous income shocks on their probability of survival and their decisions to enter export markets. In addition, since in these studies, financial frictions affect entry and export decisions directly by lowering firms' profits, not through the change in the firms' production level, the firms' selection into the domestic and foreign markets in these models only reinforces that of the Melitz (2003) model. In my model, on the contrary, this selection may be weakened leading to the distortion that less productive firms with better access to finance may operate and/or export instead of those more productive firms that are financially constrained.

This paper is also related to the literature on financial institutions and the theory of comparative advantage.[7] This line of research finds robust evidence that countries that are financially developed tend to have comparative advantage and export more in financially dependent sectors, usually defined as sectors that rely more on external finance or have less collateralizable assets. Similarly, my model can be used to analyze the effects of financial frictions on firm dynamics within each sector which give rise to the country's comparative advantage and pattern of international trade.

I begin by presenting the evidence on the positive relation between the firms' financial positions and export behaviors. The rest of the paper is organized as follows. Section 3 develops a model of financial frictions and firms' decision to export. Section 4 contains the characterizations of an equilibrium of the model. Illustrative numerical examples of the model are presented in Section 5, and Section 6 concludes.

2 Empirical Evidence

Using the data from the World Bank Enterprise Surveys for Brazil (2003) and Chile (2004),[8] I find that in line with the prediction of the model, the financial dimension is positively correlated with a firm's participation in global markets. In particular, I find that more firms exports when they

[6]This is related to the literature on finance and product market competition. Cetorelli and Strahan (2006) find that lower bank concentration leads to more firm entry and a decline in average firm size. Dellas and Fernandes (2007) and Stebunovs (2007) have formalized the findings into a general equilibrium model.

[7]See, for instance, Beck (2002, 2003), Matsuyama (2005), Becker and Greenberg (2007), and Manova (2008a) for recent reviews of this literature.

[8]Source: The World Bank, http://www.enterprisesurveys.org/portal/.

have high net worth level or when they do not perceive access to finance as a constraint to their investment, even after controlling for their productivity levels and sectoral effects.

2.1 Data and Summary Statistics

The Enterprise Surveys conducted by the World Bank mainly cover registered firms in manufacturing and certain services industries. The topics covered in the surveys include the obstacles to doing business, infrastructure, finance, labor, corruption and regulation, contract enforcement, law and order, innovation and technology, and firm productivity. This is a very unique survey database as it contains both the firms' exporting status and the financial positions which is usually not available especially for developing countries. Table 1 shows the summary statistics of the final dataset and presents evidence of the presence of financial frictions. The construction of this dataset is described in more detail below.

Excessive loan collateral requirements are likely to constrain investment opportunities. As shown in Table 1, in both countries the majority of firms that have loans face collateral requirements, and these requirements are quite restrictive. For example, in the dataset for Brazil the percentage of firms with loans requiring collateral compared to those firms that have loans is on average 67%, and the mean value of collateral needed for a loan as a percentage of the loan amount is 123%. The corresponding values for Chile are 68% and 113%, respectively. In addition, as can be seen from Table 1, the average firm age before entering the foreign markets is quite high in both countries. There are many reasons that can explain this finding, including the presence of financial frictions as firms may not be able to finance the costs involved in exporting when they are young. The surveys also provide indicators of how firms perceive their financial environment and finance their investment. In Tables 2 and 3, I report the indicators that measure the degree to which firms perceive access and costs of finance as a constraint to investment. In the surveys, firms indicate whether they find access to finance or costs of finance as no obstacle, minor obstacle, moderate obstacle, major obstacle, or very severe obstacle. In comparisons, firms in Brazil seem to face more stringent obstacles.

To construct the final dataset, I drop non-manufacturing firms and firms that did not have complete records of variables of interest, which are those regarding the firm's export status, size, production level, and balance sheet. In addition, only the data of the most recent fiscal year is considered. Net worth level is defined as the equity from the balance sheet and only those with positive net worth are included. To construct the productivity level, I use the data on the firm's total market value of production divided by firm size. If the total market value of production is not available, I use the total value of sales. Two measures of firm size are used: the total assets from the balance sheet and the average number of permanent workers. As will be seen, the results are similar across the two measures, namely asset productivity and labor productivity, although the concept of asset productivity is more relevant for my model. Then, I remove the sectoral effects that may dictate how productive firms are by running the following regression for each concept of productivity:

$$\text{productivity}_i = b_0 + \sum_{j=1}^{k-1} b_j \ \text{sector}_{ij} + \varepsilon_i \tag{1}$$

where k = the number of sectors in each country.[9] and the dummy variable $\text{sector}_{ij} = 1$ if the firm i is in sector j and 0 otherwise. In this regression, b_0 is the mean productivity of the reference sector while $(b_j + b_0)$ is the mean productivity of firms in sector j. Finally, I use the productivity level that is not explained by the above regression, ε_i, as the proxy of an individual firm's productivity level and categorize them using a five point scale for productivity, based on their quintiles. Similarly, firms are also categorized into 5-quintiles of net worth levels.

2.2 Descriptive Evidence

Using this dataset, I show that even after controlling for productivity levels, firms with higher net worth or firms that are less financially constrained are more likely to become exporters. Figures 1-4 plot, for each of the five productivity bins, the ratios of exporters for two groups of firms: those within the highest two quintiles of net worth and those within the lowest two quintiles. The clear pattern observed in both countries is that the percentage of firms exporting is higher for the group with higher net worth and the difference of the exporter ratios is on average 30% and can be as high as 50%. Due to the problem that the size of net worth and the total value of assets tend to be highly correlated, I also report the ratios of exporters for two groups of firms depending on the levels of firms' perception of access to finance as an obstacle to their investment. The first group contains those firms that do not find access to finance an obstacle to investment or only find it a minor obstacle. This group is then compared to those that view access to finance as a major or very severe obstacle to investment. The results are shown in Figures 5-8. However, it is important to note that, this indicator may also be highly correlated with firm size.[10] As can be seen, similar patterns arise: firms that face less obstacles to access to finance tend to become exporters. In addition to these figures, Table 4 reports the mean age of firms starting exporting, both conditioning on the net worth level and the access to finance indicator, and shows that firms with high net worth or facing less obstacles tend to start exporting faster which complements the findings above.

In the next section, I will present the model that can capture these characteristics of the dataset in which a firm's exporting decision depends not only on its productivity level but also on its financial strength. In particular, firms with better financial positions are more likely to export.

3 Model

This is a model where two symmetric countries trade differentiated consumption goods and can borrow and lend capital, a numeraire good, with the rest of the world at the risk-free interest rate r. In each country, there is a continuum of project opportunities of mass 1 to produce consumption goods. I refer to each project opportunity as a firm. Firms are heterogeneous and monopolistically competitive. Time is discrete where $t = 0, 1, 2, ...$ In each period t, the firms have the option to operate and wheter or not to export. An operating firm produces a single product variety denoted by ω. To isolate the importance of finance in firms' export decisions, I assume that there are no additional costs involved with international trade besides fixed costs of entering an exporting market which are in units of capital.

[9] The surveys contain nine manufacturing sectors for Brazil and six for Chile

[10] Beck et al. (2005), using the World Business Environment Survey (WBES 2000) which covers over 4,000 firms in 54 countries, find that it is the smallest firms that consistently are the most constrained.

3.1 Consumers

The representative consumer owns the firms and has a C.E.S. utility function over the set of goods that are available in the country in each period and discounts time at the rate β, $0 < \beta < 1$. I assume that the preference of the consumer takes the form $\sum_t \beta^t \left(\int_{\Omega_t} c_t(\omega)^\rho d\omega \right)^{\frac{1}{\rho}}$ with $0 < \rho < 1$. $\sigma = 1/(1-\rho) > 1$ is the constant elasticity of substitution while Ω_t denotes the measure of products available in the country in period t and consists of the set of goods that are produced domestically and the set of goods that are imported. Although the consumer own the firms, I assume that saving and borrowing decisions are only done by firms. Consider the problem of the representative consumer in country j, for example. Let Ω_t^{ij} denote the set of goods produced in country i and available for consumption in country j in period t. Thus, in each period, taking prices of each good $p_t^j(\omega)$ available in the country as given, the representation consumer in country j chooses consumption of each good ω, $c_t^j(\omega)$, from the set $\Omega_t^{jj} \cup \Omega_t^{ij}$ to maximize the utility subject to the following budget constraint:

$$\int_{\Omega_t^{jj} \cup \, \Omega_t^{ij}} p_t^j(\omega) c_t^j(\omega) d\omega = I_t^j \tag{2}$$

where I_t^j denotes the aggregate disposable income of the representative consumer in country j in period t. As is well known, the solution to the consumer's problem leads to the indirect demand function for a particular variety ω of the form $c_t^j(\omega) = \frac{I_t^j}{P_t^j}\left(\frac{p_t^j(\omega)}{P_t^j}\right)^{-\sigma}$ where $P_t^j = \left[\int_{\Omega_t^{jj} \cup \, \Omega_t^{ij}} p_t^j(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$ is the composite price index in country j in period t. Equivalently, the indirect demand function implies that each firm is facing the following pricing schedule:

$$p_t^j(\omega) = \left(\frac{I_t^j}{P_t^j} \frac{1}{c_t^j(\omega)} \right)^{1/\sigma} P_t^j \tag{3}$$

Note that this pricing schedule is the channel in which this model departs from the standard models. Here, the quantity of the products that firms can offer to the markets may be restricted due to the financial frictions they are facing. In other words, the firms' pricing has to adjust according to the above schedule, depending on the amount of the products they serve the market. Therefore, in this environment the price that each firm charges may no longer be equal to a constant markup of its marginal costs as in the case of the standard model. I explain the problem of each firm in more detail next.

3.2 Firms

As mentioned before, each firm produces only a single variety; therefore, we can identify each of them by the variety it produces, denoted by ω. They are heterogeneous in two dimensions: net worth n_t and productivity z_t. I assume that at the beginning of each period, each firm is subject to the same exogenous death probability φ.

Assumption 1: $0 < \varphi < 1$

In period t, new firms draw their productivity level z_t from a distribution $g(z)$ with support Z. The incumbent and surviving firms' productivity level evolves according to the transition function $Q(z_t/z_{t-1})$, which is assumed to be identical for all firms. Capital is the only input for production and is a numeraire good. If operating, firms incur a per-period fixed cost of production of $f^d > 0$ units of capital. In addition to this fixed cost of operating, firms have to incur an extra per-period fixed cost of $f^x > 0$ if they decide to export. These fixed costs are assumed to be identical across all firms.

Each firm has an increasing returns to scale production technology, which depends on its productivity z_t and the amount of capital k_t it uses. In particular, a domestic firm operates by the following production function, $y_t = z_t \max\left\{k_t - f^d, 0\right\}$, while an exporting firm produces according to $y_t = z_t \max\left\{k_t - f^d - f^x, 0\right\}$. The level of production k_t can be financed with the firm's own net worth i_t and one-period debt b_t. Feasibility of internal financing requires that $0 \leq i_t \leq n_t$. The amount of unused net worth $(n_t - i_t)$ can be invested through financial intermediaries at the risk-free interest rate r_t. Each firm, however, is subject to a borrowing constraint that specifies a limited amount of capital that can be externally financed. In particular, I assume that this borrowing constraint takes the following form

$$b_t \leq \eta n_t$$

where $\eta > 0$ is a fixed borrowing factor, assumed identical for all firms. I think of this factor η as summarizing the degree of financial frictions in a given economy: higher η corresponds to better access to finance by all firms. Given this specification, the level of production of each firm is given by

$$k_t = i_t + b_t$$

where $0 \leq i_t \leq n_t$ and $0 \leq b_t \leq \eta n_t$.

Note that in this environment, firms face a borrowing constraint that depends only on their net worth. While this is a quite restrictive assumption, the effects of financial frictions can easily be isolated since only the firm's financial positions (as represented by its net worth level), not its productivity level, determines the amount of debt. This assumption may be reasonable if, for example, financial intermediaries cannot verify the firm's productivity level, and thus, they can only base their lending decisions on what is verifiable, the financial position of the firm. In general, financial constraints can depend on both the firm's financial position and productivity level. For example, borrowing constraints can emerge endogenously from the enforcement problem of debt contracts as that in Albuquerque and Hopenhayn (2004), or from the optimal contract in the presence of asymmetric information between the lender and the borrower as that in Clementi and Hopenhayn (2006)

The timing within each period is as follows. At the beginning of the period, firms observe their productivity level and whether they are hit by the exogenous death probability φ which forces them to exit. The surviving firms choose whether to operate in the current period and whether to enter the foreign market. If they decide to operate, they choose the level of production, the financing plan, and the distribution policy of their outputs among the markets they participate in accordingly. At the end of the period after their income is realized and debts are repaid, firms choose the level of dividend payouts and the end-of-period net worth (retained earnings) by taking into account the evolution of their productivity $Q(z_{t+1}/z_t)$ and the probability of death φ in the next period while discounting the future at the same rate β as the consumer.

3.2.1 Firm's Problem

As described above, each firm has both static and dynamic problems. The static decisions involve the production level, the pattern of financing, the export participation, and the distribution of its product while the dynamic decisions involve the policies on the dividend payouts and the end-of-period net worth. Although the model features dynamics, I will focus on stationary equilibria with time invariant levels of risk-free interest rate, aggregate price indices, aggregate income, and the distribution of firms in each country. Therefore, the time subscripts will be dropped. In what follows, I explain the problem of a firm that starts the period with net worth n and productivity level z and survives the exogenous death probability.

Firm's Recursive Problem Given $\{I, P, r\}$, the aggregate prices and incomes in the two symmetric countries, and the risk-free interest rate, a firm with (n, z) makes the operating, exporting, retained earnings, and dividends decisions to maximize its value. Omitting the aggregate variables in the notation, let $V(n, z)$ denote the value of the firm and $\Pi(n, z)$ denote its current-period net income before the dividend distribution policy. Then, the firm's recursive problem can be written as follows:

$$
\begin{aligned}
V(n,z) = \quad & \max_{n'} \quad \Pi(n,z) - n' + \beta(1-\varphi)\int V(n',z')Q(z'/z)dz' \\
& \text{s.t.} \quad 0 \leq n' \leq \Pi(n,z)
\end{aligned}
$$

where $n'(n, z)$ is the policy on the end-of-period level of net worth, and thus, this implies the dividend policy of $D(n, z) = \Pi(n, z) - n'(n, z)$. Here I assume that the dividends cannot be negative i.e. firms cannot issue new equity to the consumer. The current-period net income is given by the maximum value of the three options the firm has: not to operate but lend out its net worth at the prevailing interest rate, to operate as a domestic firm, or to operate as an exporter. In other words, the current-period net income is

$$
\Pi(n,z) = \max\{(1+r)n,\ \pi^d(n,z),\ \pi^x(n,z)\}
$$

where $\pi^d(n, z)$ denotes the net income of the firm if it serves only the domestic market and $\pi^x(n, z)$ denotes the net income if it also exports. Let $\chi^d(n, z)$ and $\chi^x(n, z)$ be the indicator functions that specify the decision rule for operating as a domestic firm or as an exporter, respectively. Thus we have,

$$
\chi^d(n,z) = \begin{cases} 1 & \text{if } \pi^d(n,z) > \max\{(1+r)n,\ \pi^x(n,z)\} \\ 0 & \text{otherwise} \end{cases}
$$

and

$$
\chi^x(n,z) = \begin{cases} 1 & \text{if } \pi^x(n,z) > \max\{(1+r)n,\ \pi^d(n,z)\} \\ 0 & \text{otherwise} \end{cases}
$$

As will be shown in the derivation of $\pi^d(n, z)$ and $\pi^x(n, z)$, financial frictions may distort the firm's operating and exporting decisions from the environment in which they faces no financial constraints. More importantly, in contrast to the previous literature, in this model financial frictions do not only alter the firm's net profits but also its scale of production. As a result, the final effect on the firms' decisions is no longer clear as there are two countervailing forces operating simultaneously. First, at any given level of aggregate prices, income levels, and interest rate, the firm may not be able to operate at the unconstrained optimal scale and, thus, may find operating and/or exporting less attractive. However, there is also a general equilibrium effect at work. As the rest of the firms in the economy are also in a similar situation, the aggregate price and aggregate income indices will adjust. Intuitively, tighter borrowing constraints for the overall economy lead to a higher aggregate price and a lower aggregate income. A higher aggregate price, on the one hand, induces the firm to enter the markets as there is a higher demand for its product. A lower aggregate income, on the other hand, decreases the consumer's demand for the firm's product. As a consequence, if the financial friction problem is severe enough and the aggregate price effect dominates the income effect, the firm may decide to enter the domestic and/or export markets even if it would not have done so in the frictionless economy. Numerical exercises in Section 5 will show this case in which worsening financial environment can lead to an entrance of unproductive firms, those that would not have entered in the better financial environment.

Next, I show the derivation of $\pi^d(n, z)$ and $\pi^x(n, z)$. Facing the aggregate income and price levels, the interest rate, and the indirect demand for its product in each market given by the pricing schedule (3), each firm chooses the level of production and the distribution of its product optimally subject to its feasibility and borrowing constraints. Equivalently, this problem can be broken down into two steps. First, for a given level of capital k, I solve for the firm's maximum sales revenue. Then, given the revenue function, I solve for the optimal level of production and financing plan. The two-step derivation is explained in more detail below.

Sales Revenue Intuitively, a domestic firm's maximum sales revenue comes from the sales of all its outputs in the domestic market at the price given by the pricing schedule (3). An exporting firm, on the other hand, will have to take into account the indirect demand function of the foreign market as well and make decisions on how to distribute its outputs between the two markets. The problems of a non-exporting firm and an exporting firm are formally written below. To simplify the notions, here I will use superscript d to index the variables in the firm's domestic market while those in the foreign market are indexed by superscript f.

Sales Revenue of Non-Exporting Firms Taking the pricing schedule from the indirect demand function in the domestic market as given, a non-exporting firm with the level of production k and productivity z maximizes the sale revenues from only selling its product in the domestic market:

$$\begin{aligned} R^d(k,z) = \quad & \max_{y^d} \quad p^d y^d \\ & \text{s.t.} \quad y^d \leq z \max\left\{k - f^d, 0\right\} \\ & \qquad\;\; p^d = \left(\frac{I^d}{P^d y^d}\right)^{1/\sigma} P^d \end{aligned}$$

Intuitively, the solution to this problem is to sell all the outputs $y^d = z \max\left\{k - f^d, 0\right\}$ at the given price. As a result, the sales revenue function of a non-exporting firm with level of production k and productivity z takes the following form:

$$R^d(k,z) = \left(\frac{I^d}{P^d}\right)^{1/\sigma} P^d \left[z \max\left\{k - f^d, 0\right\}\right]^{1-1/\sigma} \tag{4}$$

which is differentiable for $k > f^d$ and for given aggregate prices, income levels, and the risk-free interest rate, we have

$$R_1^d(k,z) > 0, \qquad R_2^d(k,z) > 0 \tag{5}$$
$$R_{11}^d(k,z) < 0, \qquad R_{12}^d(k,z) > 0, \text{and} \quad R_{22}^d(k,z) < 0 \tag{6}$$

Sales Revenue of Exporting Firms Given the indirect demand functions of its products from both the local and foreign markets, an exporting firm with the level of production k and productivity z chooses how to distribute output between the two markets to maximize its total sales revenue given by:

$$\begin{aligned} R^x(k,z) = \max_{y^d,y^f} \quad & p^d y^d + p^f y^f \\ \text{s.t.} \quad & y^d + y^f \le z \max\left\{k - f^d - f^x, 0\right\} \\ & p^d = \left(\frac{I^d}{P^d y^d}\right)^{1/\sigma} P^d \text{ and } p^f = \left(\frac{I^f}{P^f y^f}\right)^{1/\sigma} P^f \end{aligned}$$

In general, each firm divides its sales between the domestic and the foreign markets on the basis of the ratio of its country income level and aggregate price index to that of the other country. Since we focus on a symmetric two-country model where $I^d = I^f = I$ and $P^d = P^f = P$, the firm's optimal distribution policy is simply to supply both markets the same amount. In other words, we have $y^f = y^d = \frac{1}{2}y$ where $y = z\max\left\{k - f^d - f^x, 0\right\}$. In this case, the sales revenue function of an exporting firm with level of production k and productivity z is given by the following function:[11]

$$R^x(k,z) = 2\left(\frac{I}{P}\right)^{1/\sigma} P\left(\frac{y}{2}\right)^{1-1/\sigma} \tag{7}$$

for given aggregate prices, income levels, and the interest rate, the revenue function is differentiable for $k > f^d + f^x$ and we have

$$\begin{aligned} R_1^x(k,z) &> 0, \quad R_2^x(k,z) > 0 \\ R_{11}^x(k,z) &< 0, \quad R_{12}^x(k,z) > 0, \text{ and } R_{22}^x(k,z) < 0 \end{aligned} \tag{8}$$

Recall that this model abstracts from any variable costs of exporting or tariffs. If these features are present, firms will have to adjust the distribution of their products accordingly.

Next, given these revenue functions, I solve for the optimal level of production and financing plan for each exporting status.

[11] For a general case where the two countries need not to be symmetric, $y^d = \frac{1}{\left(\frac{A^d}{A^f}\right)^{-\sigma}+1} y$ and $y^f = \left(\frac{A^d}{A^f}\right)^{-\sigma} y^d = \frac{\left(\frac{A^d}{A^f}\right)^{-\sigma}}{\left(\frac{A^d}{A^f}\right)^{-\sigma}+1} y$ where $A^d \equiv \left(\frac{I^d}{P^d}\right)^{1/\sigma} P^d$ and $A^f \equiv \left(\frac{I^f}{P^f}\right)^{1/\sigma} P^f$. Thus, in this case $R^x(i+b,z)$ $= \left(A^d \left(\frac{1}{\left(\frac{A^d}{A^f}\right)^{-\sigma}+1}\right)^{1-1/\sigma} + A^f \left(\frac{\left(\frac{A^d}{A^f}\right)^{-\sigma}}{\left(\frac{A^d}{A^f}\right)^{-\sigma}+1}\right)^{1-1/\sigma}\right) y^{1-1/\sigma}$.

Net Income of Operating Firms An operating firm's net income comes from the sales revenue of its product, the sales of its undepreciated capital (at the price of 1 as capital is the numeraire good), and the return from lending unused net worth after the debt repayment obligation has been made.

Net Income of a Non-Exporting Firm $\pi^d(n,z)$: Given the aggregate income levels, price indices, and market interest rate, a non-exporting firm with net worth n and productivity level z chooses the level of investment from its own savings, i, and the amount of borrowing, b, to maximize its net income subject to its feasibility and borrowing constraints. In other words, the firm is solving the following problem:

$$\begin{aligned}\pi^d(n,z) \quad = \quad & \max_{i\geq 0, b\geq 0} R^d(i+b,z) + (1-\delta)\max\left\{i+b-f^d, 0\right\} \\ & +(1+r)(n-i)-(1+r)b \\ & \text{s.t. } i \leq n \text{ and } b \leq \eta n\end{aligned} \tag{9}$$

where the sales revenue function $R^d(i+b,z)$ is given by (4) derived earlier. Let $i^d(n,z)$ and $b^d(n,z)$ denote the policy functions on the financing plan associated with the above problem. Then, $k^d(n,z) = i^d(n,z) + b^d(n,z)$ is the firm's resulting constrained optimal level of production. Note that if the corner solution arises, i.e. both $i^d(n,z) = 0$ and $b^d(n,z) = 0$, we have $\pi^d(n,z) = (1+r)n$, its reservation value from just lending out all its net worth.

Net Income of an Exporting Firm $\pi^x(n,z)$: Similarly, given the aggregate income levels, prices indices, and interest rate, an exporting firm with net worth n and productivity level z chooses the production level and financing plan to solve the following problem:

$$\begin{aligned}\pi^x(n,z) \quad = \quad & \max_{i\geq 0, b\geq 0} R^x(i+b,z) + (1-\delta)\max\left\{i+b-f^d-f^x, 0\right\} \\ & +(1+r)(n-i)-(1+r)b \\ & \text{s.t. } i \leq n \text{ and } b \leq \eta n\end{aligned} \tag{10}$$

where the sales revenue function $R^x(i+b,z)$ is given by (7). Let $i^x(n,z)$ and $b^x(n,z)$ denote the policy function associated with the above problem; then the firm's constrained optimal level of production is $k^x(n,z) = i^x(n,z) + b^x(n,z)$. Again, in the case that the corner solution arises, i.e. both $i^x(n,z) = 0$ and $b^x(n,z) = 0$, we have $\pi^x(n,z) = (1+r)n$.

This completes the descriptions of a firm's problem. The solutions to problems (9) and (10) above will be characterized in more detail in Section 4. As will be seen, with borrowing constraints the direct relationship between the firm's price and marginal costs (and thus the productivity level) may no longer hold. In particular, a constrained firm cannot attain the optimal level output and must charge a higher price, given by its pricing schedule. Thus, it is possible that an unconstrained but less productive firm may be able to charge a lower price than another firm that is more productive but financially constrained.

Before I define an equilibrium for this economy, I consider the rest of the world with which firms can borrow and lend capital. The rest of the world is assumed to have large enough endowments in capital that they can lend at any amount demanded by the firms in the two countries at the risk-free interest rate r. They are also assumed to have linear preferences in capital and discount the future at the rate β. Due to these assumptions, the risk-free interest rate is pinned down to $r = 1/\beta - 1$. Thus, with the positive exogenous death probability for firms $\varphi > 0$ given by Assumption 1, we have the following condition:

$$\frac{1}{\beta} > (1-\varphi)(1+r). \tag{11}$$

This condition will guarantee that there is an upper bound on the amount of net worth a firm will accumulate. In particular, when a firm has accumulated a large enough net worth, the additional benefits of net worth in terms of relaxing the borrowing constraint vanish, and the firm will start distributing dividends.

Also, given the structure of the firms' evolution, entry, and exit, the representative consumer's disposable income must be defined carefully. In particular, since the consumer owns all firms in the country, the consumer's income comes from the dividend payouts of the surviving firms. However, the consumer must also cover the aggregate net worth given to the new entrants. Thus, the disposable income of the representative consumer in country j is given by

$$I^j = \int\limits_{\Omega^{j+}} D(\omega)d\omega - \int\limits_{E^j} n(\omega)d\omega \tag{12}$$

where Ω^{j+} denotes the set of firms that survive the death shocks and E^j denotes the set of the new entrants. Recall that in this model, the measure of the exiting firms is equal to φ. We now define an equilibrium for the model.

3.3 Equilibrium

I consider a stationary equilibrium in which aggregate prices, aggregate income levels, and distributions of firms in each countries are constant overtime. A *recursive equilibrium* for a symmetric two-country model is defined by

- the policy functions of firms $\chi^d(n,z)$, $\chi^x(n,z)$, $i^d(n,z)$, $i^x(n,z)$, $b^d(n,z)$, $b^x(n,z)$, and $n'(n,z)$;
- the distribution of firms $G(n,z)$;
- the mass of new entrants $E(G(n,z))$;
- the aggregate price P and aggregate income I; and
- the interest rate r

such that in each country

1. taking as given (P, I, r) and the indirect demand functions that come from solving the consumer's problem, the firms' policy functions satisfy their optimization problem; consumer maximizes utility subject to (12);

2. the distribution of firms and the aggregate variables are consistent with individual firms' decisions and the entry and exit rules;

3. the mass of new entrants is equal to the mass of firms exiting exogenously by death shocks; and

4. the interest rate $r = 1/\beta - 1$.

Definition of a *stationary equilibrium*: With a time invariant level of risk-free interest rate, a stationary equilibrium for this economy is an equilibrium as defined above in which the aggregate price index, aggregate income, and the distribution of firms in each country are constant over time.

To illustrate the distortionary effect associated with financial frictions, I first consider the one-period version of the model and compare the firms' decisions to those in the unconstrained environment where firms are not limited on how much they can borrow. This is the case where η equals to ∞; thus, the borrowing constraint is never binding and can be dropped out of the firms' problems.

4 Characterizations of Equilibrium

First, I show that the one-period and unconstrained version of the model collapses to an equivalence of the Melitz (2003) model in which firms' operating and exporting decisions only depend on their productivity level. In particular, the model can be solved analytically for certain distribution function of firms over (n, z). Then, I characterize the firms' decisions in the environment with financial frictions.

4.1 One-Period Version of the Model

In this environment, firms will choose not to hold any net worth at the end of the period $n'(n, z) = 0$ and pay out all their income as dividends $D(n, z) = \Pi(n, z)$. Thus, trivially we have the value of the firm $V(n, z) = \Pi(n, z)$. Since this is a one-period version, I consider the case that there is no exogenous death probability, and thus, no notion of new entrants at the end of the period. The distribution of firms over (n, z) is given exogenously and is assumed that n and z are independently Pareto distributed with the same Pareto index γ. Therefore, in this case, the density function of firms of the type (n, z) is given by

$$G(n, z) = g(n)g(z)$$

where $g(n) = \gamma \frac{(n_m)^\gamma}{n^{\gamma+1}}$, $g(z) = \gamma \frac{(z_m)^\gamma}{z^{\gamma+1}}$, and $n_m > 0$ and $z_m > 0$ are the minimum support of n and z, respectively. Next, I derive the unconstrained equilibrium and use it as a benchmark for comparisons with the equilibrium with borrowing constraints.

4.1.1 Unconstrained Environment

Let $\overline{\pi}^d(n, z)$ and $\overline{\pi}^x(n, z)$ denote the unconstrained net income of the non-exporting and exporting firms, respectively. Since firms are not limited on how much they can borrow, the problem of each firm (n, z) can be rewritten as follows:

$$V(n, z) = \Pi(n, z) = \max\{(1 + r)n,\ \overline{\pi}^d(n, z),\ \overline{\pi}^x(n, z)\}$$

where

$$\begin{aligned}
\overline{\pi}^d(n,z) &= \max_{n\geq\ i\ \geq 0,\ b\geq 0} R^d(i+b,z) + (1-\delta)\max\left\{i+b-f^d,0\right\} \\
&\quad +(1+r)(n-i)-(1+r)b;\quad \text{and} \\
\overline{\pi}^x(n,z) &= \max_{n\geq\ i\ \geq 0,\ b\geq 0} R^x(i+b,z) + (1-\delta)\max\left\{i+b-f^d-f^x,0\right\} \\
&\quad +(1+r)(n-i)-(1+r)b
\end{aligned}$$

and $R^d(i+b,z)$ and $R^x(i+b,z)$ are given by (4) and (7).

As shown in Proposition 1, in this environment, each firm always operates at the unconstrained optimal scale and its decisions to operate and/or export will only depend on its productivity, independent of its net worth. Therefore, the model reduces to that similar to Melitz (2003). In proving Proposition 1, Assumption 2 is needed for the aggregate income and prices to be finite while Assumptions 3 and 4 regarding the fixed costs guarantee that the cutoffs for operating and exporting be binding, since then not all firms choose to operate and export in equilibrium.

Assumption 2: $\sigma - \gamma - 1 < 0$.

Assumption 3: $2f^d > N$ where $N = \int n(\omega)d\omega$.

Assumption 4: $f^x > f^d$.

Let $\underline{z}^{FB}$ denote the equilibrium productivity cutoff level for operating decisions and $\overline{z}^{FB}$ denote the corresponding equilibrium productivity cutoff level for exporting. Then, by Proposition 1, firms with high enough productivity, i.e. $z \geq \underline{z}^{FB}$, will enter the domestic market while those very productive firms, $z \geq \overline{z}^{FB}$, will also export.

Proposition 1. *Given interest rate r, a firm's operating and exporting decisions in the unconstrained environment depend only on its productivity level, and the decisions are characterized by two cutoffs $\underline{z}^{FB}$ and $\overline{z}^{FB}$ where all firms with $z \geq \underline{z}^{FB}$ operate and those with $z \geq \overline{z}^{FB}$ export and $z_m < \underline{z}^{FB} < \overline{z}^{FB} < \infty$. In particular,*

$$\begin{aligned}
\underline{z}^{FB} &= z_m \left[f^d\right]^{1/(\sigma-1)} \left\{ \frac{1-1/\sigma-\gamma}{\sigma-\gamma-1} \frac{\left[\left[f^d\right]^{(\sigma-\gamma-1)/(\sigma-1)} + \left[f^x\right]^{(\sigma-\gamma-1)/(\sigma-1)}\right]}{N} \right\}^{1/\gamma} \quad \textit{and} \\
\overline{z}^{FB} &= z_m \left[f^x\right]^{1/(\sigma-1)} \left\{ \frac{1-1/\sigma-\gamma}{\sigma-\gamma-1} \frac{\left[\left[f^d\right]^{(\sigma-\gamma-1)/(\sigma-1)} + \left[f^x\right]^{(\sigma-\gamma-1)/(\sigma-1)}\right]}{N} \right\}^{1/\gamma}
\end{aligned}$$

As can be seen, both cutoffs are independent of the interest rate r while they are decreasing in N, the aggregate amount of net worth in the country. Given the functional form above, the number of varieties increases with the size of the economy, but not linearly, due to the factor γ. With this result, the optimal decision rules for operating as a domestic firm and as an exporter are given by the following indicator functions:

$$\chi^d(n,z) = \begin{cases} 1 & \text{if } \underline{z}^{FB} \leq z < \overline{z}^{FB} \\ 0 & \text{otherwise} \end{cases}$$

and

$$\chi^x(n,z) = \begin{cases} 1 & \text{if } z \geq \overline{z}^{FB} \\ 0 & \text{otherwise} \end{cases}$$

Appendix 3.A presents the proof of the Proposition as well as the derivations of the unconstrained production levels ($\overline{k}^d(z)$, $\overline{k}^x(z)$), price functions ($\overline{p}^d(z)$, $\overline{p}^x(z)$), sales revenue functions ($\overline{R}^d(z)$, $\overline{R}^x(z)$), all of which are independent of n, and the value functions ($\overline{\pi}^d(n,z)$, $\overline{\pi}^x(n,z)$) of a non-exporting firm and an exporting firm, respectively. Next, I derive the firms' decision rules in an environment with borrowing constraints.

4.1.2 With Borrowing Constraints

In this case, I show that not only the productivity level matters for the firms' operating and exporting decisions, but the net worth level also plays an important role. For example, although there is a minimum productivity level below which firms will not find operating profitable, this will no longer guarantee that firms with productivity higher than this level will actually operate. In order to operate, they also need to have a high enough net worth. In other words, there is another dimension of cutoff decision rules in terms of net worth that a firm will take into account. As will be seen, for a given productivity, firms with high net worth are more likely to become exporters similar to what is found in Section 2.

Formally, let $\underline{z}$ denote the minimum cutoff productivity level for operating, i.e. $\underline{z} = \inf\{z : \pi^d(n,z) > \max\{(1+r)n,\ \pi^x(n,z)\}$ for all $n\}$. Thus, firms that are not productive enough with $z < \underline{z}$ will not find operating profitable no matter how much net worth they have. However, for those $z > \underline{z}$, the decision whether or not to operate will also depend on the firm's net worth. Note that $\underline{z}$ is generally different from $\underline{z}^{FB}$ since the equilibrium aggregate price and income may differ across the two environments. Let $\underline{n}^d(z)$ denote this threshold level of net worth below which firms will not be able to operate profitably. In particular, for $z > \underline{z}$, $\underline{n}^d(z) = \inf\left\{n \geq 0 : \pi^d(n,z) > \max\{(1+r)n,\ \pi^x(n,z)\}\right\}$ and, therefore, for those firms with for $z > \underline{z}$ and $n \leq \underline{n}^d(z)$, $\pi^d(n,z) = (1+r)n$.

Similarly, in this environment there will be a cutoff productivity level for exporting which, again, can be different from $\overline{z}^{FB}$. Let $\overline{z}$ denote this cutoff level and $\overline{z} = \inf\{z : \pi^x(n,z) > \max\{(1+r)n,\ \pi^d(n,z)\}$ for all $n\}$ and, thus, firms with $z < \overline{z}$ will not export regardless of their net worth levels. Like the case of the entry decision into the domestic market above, there will be another cutoff in terms of net worth which will specify whether firms will find exporting profitable. This net worth threshold for $z > \overline{z}$ is defined as $\underline{n}^x(z) = \inf\left\{n \geq 0 : \pi^x(n,z) > \max\{(1+r)n,\ \pi^d(n,z)\}\right\}$.

Intuitively, with borrowing constraints both net worth and productivity determine the operating and exporting decisions. On the one hand, firms of a given productivity choose to operate and/or export if they are large enough to run their business at a profitable scale. On the other hand, firms of a given size choose to operate and/or export if they are productive enough. With the definitions of all the cutoffs both in terms of net worth and productivity above, we can write the decision rules for operating as a domestic firm and as an exporter as follows:

$$\chi^d(n,z) = \begin{cases} 1 & \text{if } (1)\underline{z} \leq z < \overline{z} \text{ and } n \geq \underline{n}^d(z) \text{ or } (2)\ z \geq \overline{z} \text{ and } \underline{n}^d(z) \leq n < \underline{n}^x(z) \\ 0 & \text{otherwise} \end{cases}$$

and

$$\chi^x(n,z) = \begin{cases} 1 & \text{if } z \geq \overline{z} \text{ and } n \geq \underline{n}^x(z) \\ 0 & \text{otherwise} \end{cases}$$

In Proposition 2, I show that besides the thresholds $\underline{n}^d(z)$ and $\underline{n}^x(z)$ above, there are two additional thresholds that specify the required minimum levels of net worth to achieve the unconstrained efficient scale of operating and exporting, thus, allowing firms to attain the unconstrained values of net income $\overline{\pi}^d(n,z)$ and $\overline{\pi}^x(n,z)$, respectively. Let $\overline{n}^d(z)$ denote that for a non-exporting firm and is defined as $\overline{n}^d(z) = \inf\{n \geq 0 : i^d(n,z) + b^d(n,z) = \overline{k}^d(z)\}$ for $z > \underline{z}$. Similarly, let $\overline{n}^x(z)$ denote the net worth level to achieve the unconstrained efficient scale for an exporting firm and is defined as $\overline{n}^x(z) = \inf\{n \geq 0 : i^x(n,z) + b^x(n,z) = \overline{k}^x(z)\}$ for $z > \overline{z}$. Therefore, potentially there are five groups of firms in equilibrium on the space of (n,z): non-operating firms, unconstrained and constrained non-exporting firms, and unconstrained and constrained exporting firms.

Before we derive the properties of these four thresholds of net worth in Proposition 2, Lemma 1 derives properties of the firms' production levels and net-income functions.

Lemma 1. *For* $0 < \eta < \infty$,

(i) $i^d(n,z) = n$, $b^d(n,z) = \eta n$, *and* $\pi^d(n,z)$ *is strictly increasing and strictly concave in* n *on* $[\underline{n}^d(z), \overline{n}^d(z))$ *for all* $z > \underline{z}$;

(ii) $i^x(n,z) = n$, $b^x(n,z) = \eta n$, *and* $\pi^x(n,z)$ *is strictly increasing and strictly concave in* n *on* $[\underline{n}^x(z), \overline{n}^x(z))$ *for all* $z > \overline{z}$.

Lemma 1 states that it is optimal for a constrained firm, be it a non-exporting firm or an exporting firm, to invest all its net worth in their production and to borrow up to the limit. In particular, in the proof of Lemma 1, I establish that $\pi_1^d(n,z)$ and $\pi_1^x(n,z)$ are strictly higher than $(1+r)$ for a financially constrained firm and approaching $(1+r)$ as net worth approaches $\overline{n}^d(z)$ and $\overline{n}^x(z)$, for a domestic firm and an exporting firm respectively. This is a usual result of models with financial frictions as there is an extra incentive for firms to save to overcome their financial constraints.

Proposition 2. *For* $0 < \eta < \infty$, *the four thresholds of net worth have the following properties:*

(i) for all $z > \underline{z}$, $\overline{n}^d(z) = \frac{\overline{k}^d(z)}{1+\eta}$ *and* $\overline{n}^d(z)$ *is strictly increasing in* z;

(ii) for all $z > \underline{z}$, $\underline{n}^d(z) \in \left(\frac{f^d}{1+\eta}, \overline{n}^d(z)\right)$ *and* $\underline{n}^d(z)$ *is strictly decreasing and strictly convex in* z;

(iii) for all $z > \overline{z}$, $\overline{n}^x(z) = \frac{\overline{k}^x(z)}{1+\eta}$ *and* $\overline{n}^x(z)$ *is strictly increasing in* z;

(iv) for all $z > \overline{z}$, $\underline{n}^x(z) \in \left(\underline{n}^d(z), \overline{n}^x(z)\right]$ *is strictly decreasing in* z.

The proofs of Lemma 1 and Proposition 2 are shown in Appendix 3.A. Given the above results, if $z_m < \underline{z} < \overline{z}$ i.e. the productivity cutoffs are binding, we have five different groups of firms in equilibrium as illustrated in Figure 9. As can be seen from the figure, conditioning on net worth, the propensity to operate and/or export increases with productivity and similarly, conditioning on productivity, the propensity to operate and export increases with net worth levels. However, the distortionary effects of borrowing constraints, which is the case when $\underline{z} < \underline{z}^{FB}$ and $\overline{z} < \overline{z}^{FB}$, cannot be proven analytically. This can be seen since we do not have the analytical functions of $\underline{n}^d(z)$ and $\underline{n}^x(z)$ and, thus, we do not have the closed-form functions of the aggregate prices and income levels taking into account the decision rules of all firms. However, it is likely that a distortion will occur if the aggregate price effect dominates the aggregate income effect. Intuitively, financial frictions constrain firms' scale of operation and force them to charge higher prices. Overall, aggregate price tends to increase while aggregate income declines. When the price effect dominates, cutoffs move left and less productive firms are able to enter profitably. Next, I characterize the dynamic model using the firm's optimal static decision rules above.

4.2 Dynamic Model

Given the optimal static entry and exporting decisions above, the dynamic problem of a firm with (n, z) can be rewritten as follows:

$$\begin{aligned} V(n,z) = \quad & \max_{n'} \quad \Pi(n,z) - n' + \beta\,(1-\varphi) \int V(n',z')Q(z'/z)dz' \\ & \text{s.t.} \quad 0 \leq n' \leq \Pi(n,z) \end{aligned}$$

where $\Pi(n, z)$ has the following properties:

$$\Pi(n,z) = \begin{cases} \pi^x(n,z) & \text{if } z \geq \overline{z} \text{ and } n \geq \underline{n}^x(z) \\ \pi^d(n,z) & \text{if (1) } \underline{z} < z \leq \overline{z} \text{ and } n \geq \underline{n}^d(z) \text{ or (2) } z \geq \overline{z} \text{ and } \underline{n}^d(z) \leq n < \underline{n}^x(z) \\ (1+r)n & \text{otherwise} \end{cases} \tag{13}$$

With the above functional form of $\Pi(n, z)$ and the proof of Lemma 1, we have $\Pi_1(n, z) \geq (1+r)$. As can be seen above, for non-operating firms, $\Pi_1(n, z) = (1 + r)$. However, this is also true for unconstrained firms as for these firms there is no extra benefit of accumulating net worth exceeding $\overline{n}^d(z)$ or $\overline{n}^x(z)$. These firms can operate at their efficient scale and, therefore, their extra net worth would only be invested at the risk-free interest rate. Financially constrained firms, on the contrary, will have an extra incentive to save as the additional net worth that they accumulate will help relax their borrowing constraint in the next period. Specifically, as shown in Appendix 3.A, for constrained domestic firms $\Pi_1(n, z) = \pi_1^d(n, z) = (1 + \eta) \left[R_1^d((1 + \eta)\, n, z) + (1 - \delta) - (1 + r)\right] + (1 + r) > (1 + r)$ since $(1 + \eta)\, n < \overline{k}^d(z)$ and, similarly, for constrained exporters $\Pi_1(n, z) = \pi_1^x(n, z) = (1 + \eta) \left[R_1^x((1 + \eta)\, n, z) + (1 - \delta) - (1 + r)\right] + (1 + r) > (1 + r)$.

In what follows, I show that the deterministic version of the model, in which firms' productivity is constant over time, can feature the persistent distortion of financial frictions on firms' entry decisions: productive firms that do not have enough initial net worth will follow the path of disaccumulation of net worth and, as a result, they will eventually not operate and thus, not export. This is quite a striking result in that it differs from the usual view of firms always saving their way out of the borrowing constraints. To put it differently, there can be another threshold of net worth such that firms with net worth below this level will choose to disaccumulate net worth at the end of the period and eventually they will reach the state where they do not hold any net worth. As a result, given the form of the borrowing constraints they are facing, will stay in this state forever i.e. they will eventually not operate and/or export. Thus, this model features the poverty trap issue and this may be due to the fact that the discounted benefits of accumulating net worth is lower than the cost of interest or it takes too long for a firm to accumulate asset out of the credit constraints. For example, in the next section, we will see that in the deterministic case there exists at least a threshold of net worth, such that when firms start the period with net worth low enough, these firms will optimally choose to disaccumulate their net worth. Moreover, this threshold is strictly decreasing and that the more restrictive the borrowing constraint (the lower the η), the higher the threshold.

5 Numerical Exercises

To illustrate the model's long-term effects of borrowing constraints, I use the concept of a stationary equilibrium. In particular, I perform numerical exercises by varying the degree of the tightness of the borrowing constraints η.

5.1 Parametrization

Currently, the model has not been calibrated to any particular country. The following are standard parameter values.

Parameter	Description	Comments
$\beta = 0.956$	Intertemporal discount rate	Cooley and Quadrini (2001)
$\rho = 0.6$	So that elasticity of substitution $\sigma = 2.5$	
$\delta = 0.07$	Depreciation rate	Cooley and Quadrini (2001)
$\varphi = 0.05$	Probability of exogenous exit	Exit rate 3-7% per year Bartelsman et al. (2005)
$\eta = 3$ (benchmark)	Borrowing constraint factor	Buera and Shin (2007)

The rest of the parameters are ones that in the stationary equilibrium of the benchmark model, the productivity cutoffs are binding and the ratio of the fixed costs of operating to the aggregate capital is approximately 4%.

Parameter	Description	Comments
$f^d = 10$	Fixed costs of operating	
$f^x = 20$	Fixed costs of exporting	Chosen relative to f^d
$E(G(n, z))$ uniformly distributed	Distribution of new entrants over (n, z)	

5.2 Illustrative Numerical Examples: Deterministic Dynamic Model

In this section, I consider the deterministic version of the model and compare the results of the benchmark model with $\eta = 3$ to those of the case with $\eta = 0.8$ (corresponding to the value of Brazil in Section 2). Figures 10 - 14 present the results of the numerical exercises. Panel a) of each figure presents the results for the benchmark model whereas panel b) corresponds to those of the environment with tighter borrowing constraints.

Figure 10 shows the firms' entry and export decision rules which confirm that with borrowing constraints both net worth and productivity affect the firms' decisions. In particular, for a given level of net worth, firms with high enough productivity will enter and those that are even more productive will export. By the same token, for a given productivity level, firms that have high net worth will choose to operate and those with even higher levels of net worth will be more likely to export. By comparing Figure 10 panels a) and b), we see that a reduction of η from 3 to 0.8 leads to a distortionary effect. From the figure, the cutoffs of productivity for entry and export decisions, $\underline{z}$ and $\overline{z}$, are lower with $\eta = 0.8$, implying that, in this case, less productive firms can enter the domestic market and/or the foreign market profitably, although they would not have found entry and export profitable in the less restrictive environment. On the contrary, the threshold levels of net worth required for entry and exporting, $\underline{n}^d(z)$ and $\underline{n}^x(z)$, are higher in panel b). This is intuitive, as the lower the borrowing limit, the higher the net worth needed to finance the fixed costs.

Figure 11 elaborates on Figure 10 by indicating the financial conditions of each operating firm in addition to the entry and exporting decisions. In both panels, the five regions of (n, z) similar to those in Figure 9 are presented. For firms with the same exporting status, those with high net worth are more likely to operate on a more efficient scale. More importantly, with tighter borrowing constraints firms require much higher net worth to achieve the efficient scale. That is, we have $\overline{n}^d(z)$ and $\overline{n}^x(z)$ shift up with lower η.

Figure 12 illustrates how firms distribute their current-period net income $\Pi(n, z)$ between the end-of-period net worth $n'(n, z)$ and the dividend payouts $D(n, z)$. Within each panel, the $\Pi(n, z)$, $n'(n, z)$, and $D(n, z)$ functions are plotted for three groups of firms: those with the lowest productivity level z_1, those with medium-range productivity $z_{nz/2}$, and those most productive firms z_{nz}. The first similarity between the two panels is that the net income function is increasing in both arguments. Also, the least productive firms do not accumulate any net worth at the end of the period, thus, distributing all income as dividends. Firms with higher productivity, on the contrary, tend to accumulate more net worth, thus, not distributing dividends right away. This is with the exception of firms with sufficiently low net worth, represented by those at the lower left corner of each diagram. This is surprising and suggests that financial frictions can have persistent effects on firms' decisions to enter the market. As discussed in Section 4.2, with firms discounting the future heavily enough, it may be the case that they optimally choose not to save, as saving involves too much of a sacrifice. When comparisons are made between the two panels, we see that the most productive firms with high net worth are actually doing better with more restrictive borrowing conditions. The intuition for this is that with lower η, the firm's competitors are doing much worse and while they are forced to charge a higher price, there will be more demand for the firm's product. Also, it is clear from the figure that with lower η, firms have to save more in order to overcome the borrowing constraints and, thus, they will start paying out dividends later. This also implies that there is a larger dispersion of firm size as compared to the case with higher η. In other words, with tighter borrowing constraints, the largest firms are much larger than the smallest ones.

Figure 13 further elaborates on Figure 12 presenting the saving policy functions for all firms over the (n, z) space. In particular, they show the existence of the poverty trap issue as there are firms that do not disaccumulate assets at the end of the period, even for those that are very productive. Figure 14 plots the dynamics of net worth accumulation for three groups of firms with different productivity levels. The intersection between the firms' saving policy function and the 45-degree line indicate the threshold of net worth below which firms will disaccumulate net worth and thus will eventually reach the state where they have no assets. As can be seen this threshold decreases with productivity level while increasing with η.[12]

6 Concluding Remarks

This paper analyzes the effects of financial market imperfections on a firm's operating and exporting decisions. I develop a trade model with heterogeneous firms facing borrowing constraints in financing their production. This framework is able to capture the positive relationship between entry into foreign markets and firms' financial strengths as often seen as evidence in favor of finan-

[12] This result is supported by the evidence on the contribution of the small and medium enterprise (SME) sector. Ayyagari et al. (2007) find that low entry costs, easy access to finance (low costs of property registration which ease the collateralization process) and greater information sharing predict a large SME sector in manufacturing.

cial constraints. In particular, this paper addresses the relationship by modeling the endogenous determination of firms' financial positions and export participation. The model also provides an understanding of how entry and exporting decisions depend on firms' financial positions and provides insights on the costs of financial frictions to countries in terms of the misallocation of resources away from productive firms.

References

[1] Albuquerque, R. and Hopenhayn, H. A. (2004). "Optimal Lending Contracts and Firm Dynamics". *Review of Economic Studies*, 71:2, 285-315.

[2] Alessandria, G. and Choi, H. (2007). "Establishment Heterogeneity, Exporter Dynamics, and the Effects of Trade Liberalization". Federal Reserve Bank of Philadelphia.

[3] Ayyagari, M., Beck, T., and Demirguc-Kunt, A. (2007). "Small and Medium Enterprises across the Globe". *Small Business Economics*, 29:4, 415-434

[4] Bartelsman, E., Scarpetta, S., and Schivardi, F. (2005). "Comparative Analysis of Firm Demographics and Survival: Evidence from Micro-Level Sources in OECD Countries ". *Industrial and Corporate Change*, 14:3, 365-391.

[5] Beck, T. (2002). "Financial Development and International Trade. Is There a Link?". *Journal of International Economics*, 57:1, 107-131.

[6] Beck, T. (2003). "Financial Dependence and International Trade". *Review of International Economics*, 11:2, 296-316.

[7] Beck, T., Demirguc-Kunt, A. and Maksimovic, V. (2005). "Financial and Legal Constraints to Growth: Does Firm Size Matter?". *The Journal of Finance*, 60:1, 137-177.

[8] Becker, B. and Greenberg, D. (2007). "Financial Development, Fixed Costs and International Trade". University of Illionois at Urbana-Champaign.

[9] Bernard, A. B. and Jensen, J. B. (2004). "Why Some Firms Export". *The Review of Economics and Statistics*, 86:2, 561-569.

[10] Bernard, A. B., Eaton, J., Jensen, J. B., and Kortum, S. (2003). "Plants and Productivity in International Trade". *The American Economic Review*, 93:4, 1268-1290.

[11] Buera, F. J. (2008). "A Dynamic Model of Entrepreneurship with Borrowing Constraints: Theory and Evidence". Northwestern University.

[12] Buera, F. J. and Shin, Y. (2008). "Financial Frictions and the Persistence of History: A Quantitative Exploration". Northwestern University and University of Wisconsin.

[13] Campa, J. M. and Shaver, J. M. (2002). "Exporting and Capital Investment: On the Strategic Behavior of Exporters". IESE Business School.

[14] Cetorelli, N. and Strahan, P. E. (2006). "Finance as a Barrier to Entry: Bank Competition and Industry Structure in Local U.S. Markets". *Journal of Finance*, 61:1, 437-461.

[15] Chaney, T. (2005). "Liquidity Constrained Exporters". University of Chicago.

[16] Clementi, G. L. and Hopenhayn, H. (2006). "A Theory of Financing Constraints and Firm Dynamics". *Quarterly Journal of Economics*, 121:1, 229-265.

[17] Cooley, T. F. and Quadrini, V. (2001). "Financial Markets and Firm Dynamic". *American Economic Review*, 91:5, 1286-310.

[18] Costantini, J. A. and Melitz, M. J. (2007). "The Dynamics of Firm-Level Adjustment to Trade Liberalization". INSEAD and Princeton University.

[19] Dellas, H. and Fernandes, A. (2007). "Finance and Competition". University of Bern.

[20] Garcia-Vega, M. and Guariglia, A. (2007). "Volatility, Financial Constraints, and Trade". University of Nottingham.

[21] Greenaway, D., Guariglia, A. and Kneller, R. (2007). "Financial Factors and Exporting Decisions". *Journal of International Economics*, 73:2, 377-395.

[22] Hopenhayn, H. (1992). "Entry, Exit, and Firm Dynamic in Long Run Equilibrium". *Econometrica*, 60:5, 1127-1150.

[23] Kletzer, K. and Bardhan, P. (1987). "Credit Markets and Patterns of International Trade". *Journal of Development Economics*, 27:1-2, 57-70.

[24] Levine, R. (2005). "Finance and Growth: Theory and Evidence". In: Aghion, P and Durlauf, S. (eds.), Handbook of Economic Growth. Elsevier Science. The Netherlands.

[25] Manova, K. (2008a). "Credit Constraints, Equity Market Liberalizations and International Trade". *Journal of International Economics*, 76:1, 33-47.

[26] Manova, K. (2008b). "Credit Constraints, Heterogeneous Firms, and International Trade". Stanford University.

[27] Matsuyama, K. (2005). "Credit Market Imperfections and Patterns of International Trade and Capital Flows". *Journal of the European Economic Association*, 3:2-3, 714-723.

[28] Melitz, M. (2003). "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity". *Econometrica*, 71:6, 1695-1725.

[29] Rajan, R. and Zingales, L. (1998). "Financial Development and Growth". *American Economic Review*, 88:3, 559-586.

[30] Roberts, M. and Tybout, J. (1999). "An Empirical Model of Sunk Costs and the Decision to Export". Policy Research Working Paper Series 1436, The World Bank.

[31] Stebunovs, V. (2007). "Finance as a Barrier to Entry: U.S. Bank Deregulation and Volatility". Boston College.

[32] Zia, B. H. (2005). "The Effectiveness and Allocation of Subsidized Credit: Evidence from Export Loans". MIT.

Appendix 3.A: Proofs

Proof of Proposition 1

Non-Exporting Firms: Since in the absence of borrowing contraints i and b enter the problem of a non-exporting firm in the same way, the net income and thus the value of a non-exporting firm is equivalent to $\overline{V}^d(n,z) = \overline{\pi}^d(n,z) = \max_{k\geq 0} R^d(k,z) + (1-\delta)\max\left\{k - f^d, 0\right\} - (1+r)k + (1+r)n$ where $k = i + b$. Let $\overline{k}^d(n,z) \equiv \arg\max_{k\geq 0} \overline{V}^d(n,z)$ denote the unconstrained first-best production level. From the first-order condition that characterizes the solution to this problem, we have $R_1^d(k,z) = (\delta + r)$. After using the sales revenue function (4) and to simplify the notation let $A \equiv \left(\frac{I}{P}\right)^{1/\sigma} P$, the unconstrained first-best production level of a non-exporting firm with (n,z) turns out to depend only on the productivity level z and independent of the net worth. Thus, we can rewrite $\overline{k}^d(z)$ instead of $\overline{k}^d(n,z)$ and in particular we have

$$\overline{k}^d(z) = \left[\frac{A\left(1-1/\sigma\right) z^{1-1/\sigma}}{\delta + r}\right]^{\sigma} + f^d \tag{14}$$

Thus, $\overline{k}^d(z)$ is increasing in z, but the curvature depends on the size of the elasticity of substitution σ. In particular, $\overline{k}^d(z)$ is strictly convex in z if $\sigma > 2$, linear if $\sigma = 2$, and strictly concave if $2 > \sigma > 1$. Note that although the optimal level of production $\overline{k}^d(z)$ is uniquely pinned down, any financing plan such that $i^d(n;z) + b^d(n;z) = \overline{k}^d(z)$ and $i^d(n;z) \leq n$ is feasible. As a result, the price level associated with the first-best level of production and thus the sales revenue will also turn out to be independent of net worth. Let $\overline{p}^d(z)$ denote that price and $\overline{R}^d(z)$ denote the sales revenue function, then from the indirect demand function, we have

$$\overline{p}^d(z) = A\left(\frac{1}{z\left(\overline{k}^d(z) - f^d\right)}\right)^{1/\sigma} = \frac{\delta + r}{\left(1 - 1/\sigma\right) z} = \frac{\delta + r}{\rho z} \text{ and} \tag{15}$$

$$\overline{R}^d(z) = \left[A z^{1-1/\sigma}\right]^{\sigma}\left[\frac{1-1/\sigma}{\delta + r}\right]^{\sigma - 1} \tag{16}$$

Thus, more productive firms are charging lower prices and the price is a constant markup of the marginal cost $1/z$. Note that this is similar to that in Melitz (2003); with labor as the only input of production, $\overline{p}^d(z) = \frac{1}{\rho z}$. Now we can rewrite the firm's value function as follows

$$\overline{V}^d(n,z) = A^{\sigma} z^{\sigma - 1}\left[\frac{(1-1/\sigma)}{\delta + r}\right]^{\sigma - 1}\frac{1}{\sigma} + (1+r)\left(n - f^d\right) \tag{17}$$

Notice that for these firms, $\overline{k}^d(z)$, $\overline{R}^d(z)$, and $\overline{V}^d(n,z)$ are strictly increasing while $\overline{p}^d(z)$ is strictly decreasing in z.

Exporting Firms: Similarly, let $\overline{k}^x(z)$, $\overline{p}^d(z)$, $\overline{p}^f(z)$, $\overline{R}^x(z)$, and $\overline{V}^x(n,z)$ denote the corresponding unconstrained production level, prices that exporting firm charges in the domestic and foreign markets, the sales revenue and the value of the firm, respectively. Then $\overline{V}^x(n,z) = \overline{\pi}^x(n,z) =$

$\max_{k\geq 0} R^x(k,z) + (1-\delta)\max\left\{k - f^d - f^x, 0\right\} - (1+r)k + (1+r)n$ and $\overline{k}^x(z) \equiv \arg\max_{k\geq 0} \overline{V}^x(n;z)$. Using the sales revenue function for the symmetric two-country case (8), exporting firms requires the scale of operation that is larger than those non-exporting counterparts and in particular

$$\overline{k}^x(z) = 2\left[\frac{A\,(1-1/\sigma)\,z^{1-1/\sigma}}{\delta + r}\right]^{\sigma} + f^d + f^x > \overline{k}^d(z) \tag{18}$$

Similar to the case of non-exporting firms, the level of production is increasing with productivity while the curvature depends on the size of the elasticity of substitution σ. Moreover, in the symmetric case due to the absence of transactional costs of exporting, exporting firms will charge the same price in the domestic and the foreign market. Since in this case the firm supplies equal units of its product in both markets $y^x = y^d = \frac{1}{2}y$, here we have the price function

$$\overline{p}^d(z) = \overline{p}^f(z) = \frac{\delta + r}{(1-1/\sigma)\,z} \text{or } \frac{\delta + r}{\rho z} \tag{19}$$

Thus, the sales revenue function and the value function of an exporting firm are as follows

$$\overline{R}^x(z) = 2\left[Az^{1-1/\sigma}\right]^{\sigma}\left[\frac{1-\frac{1}{\sigma}}{\delta + r}\right]^{\sigma-1} > \overline{R}^d(z) \tag{20}$$

$$\overline{V}^x(n,z) = 2A^{\sigma}z^{\sigma-1}\left[\frac{(1-1/\sigma)}{\delta + r}\right]^{\sigma-1}\frac{1}{\sigma} + (1+r)\left(n - f^d - f^x\right) \tag{21}$$

To prove that given Assumptions 2-4, there are three groups of firms in equilibrium: those that do not operate, those that only serve the domestic market, and those that do export, we do it in three steps. First suppose that in equilibrium, there are two cutoffs of productivity $z_m < \underline{z}^{FB} < \overline{z}^{FB} < \infty$ such that for $z \geq \underline{z}^{FB}$, $\overline{V}^d(n;z) \geq (1+r)n$; and for $z \geq \overline{z}^{FB}$, $\overline{V}^x(n;z) \geq \overline{V}^d(n;z)$. Then, we solve for corresponding aggregate variables (I and P). In equilibrium, the aggregate variables must be consistent with the individual firms' behaviors. The last step is to verify that the cutoffs satisfy the assumed properties.

From the derivation above $\overline{V}^d(n,z) \geq (1+r)n$ if and only if $A^{\sigma}z^{\sigma-1}\left[\frac{(1-1/\sigma)}{\delta+r}\right]^{\sigma-1}\frac{1}{\sigma} - (1+r)f^d \geq 0$. Therefore, we have the cutoff of operating $\underline{z}^{FB}$ to satisfy

$$\underline{z}^{FB} = \left[\frac{(1+r)f^d}{\frac{1}{\sigma}A^{\sigma}}\right]^{1/(\sigma-1)}\left[\frac{\delta + r}{1-\frac{1}{\sigma}}\right] \tag{22}$$

Similarly, to find $\overline{z}^{FB}$, we have $\overline{V}^x(n,z) \geq \overline{V}^d(n,z)$ if and only if $A^{\sigma}z^{\sigma-1}c(\sigma,\delta,r) - (1+r)f^x > 0$. Thus, it must be the case that

$$\overline{z}^{FB} = \left[\frac{(1+r)f^x}{\frac{1}{\sigma}A^{\sigma}}\right]^{1/(\sigma-1)}\left[\frac{\delta + r}{1-\frac{1}{\sigma}}\right] \tag{23}$$

and by Assumption 4 we have $\underline{z}^{FB} < \overline{z}^{FB}$. Next, we solve for the aggregate variables which must be consistent with the individual firms' behaviors. Due to the structure of our model, this can be done by solving everything in terms the variable A defined earlier. In particular,

$$A = \left(\frac{I}{P}\right)^{1/\sigma} P = I^{1/\sigma}P^{1-1/\sigma}$$

The aggregate income I consists of the income from the three types of firms

$$I_t = \int_n \int_{z_m}^{\underline{z}^{FB}} (1+r) n g(z) g(n) dz dn + \int_n \int_{\underline{z}^{FB}}^{\overline{z}^{FB}} \overline{V}^d(n,z) g(z) g(n) dz dn + \int_n \int_{\overline{z}^{FB}} \overline{V}^x(n,z) g(z) g(n) dz dn \tag{24}$$

where it can be shown that the aggregate income of the non-operating firms

$$\int_n \int_{z_m}^{\underline{z}^{FB}} (1+r) n g(z) g(n) dz dn = (1+r) \left[\frac{\gamma n_m}{\gamma - 1}\right] \left[1 - \left(\frac{z_m}{\underline{z}^{FB}}\right)^{\gamma}\right]$$

while the aggregate income of the non-exporting firms

$$\int_n \int_{\underline{z}^{FB}}^{\overline{z}^{FB}} \overline{V}^d(n,z) g(z) g(n) dz dn = \left[\frac{1 - 1/\sigma}{\delta + r}\right]^{\sigma - 1} \frac{\frac{1}{\sigma} A^{\sigma} \gamma z_m^{\gamma}}{\sigma - \gamma - 1} \left[\left(\overline{z}^{FB}\right)^{\sigma-\gamma-1} - \left(\underline{z}^{FB}\right)^{\sigma-\gamma-1}\right] - (1+r) \left[\left(\frac{z_m}{\overline{z}^{FB}}\right)^{\gamma} - \left(\frac{z_m}{\underline{z}^{FB}}\right)^{\gamma}\right] \left[\frac{\gamma n_m}{\gamma - 1} - f^d\right]$$

and by Assumption 2 the aggregate income of the exporting firms is finite and is given by

$$\int_n \int_{\overline{z}^{FB}} \overline{V}^x(n,z) g(z) g(n) dz dn = (1+r) \left(\frac{z_m}{\overline{z}^{FB}}\right)^{\gamma} \left[\frac{\gamma n_m}{\gamma - 1} - f^d - f^x\right] - 2 \left[\frac{1 - 1/\sigma}{\delta + r}\right]^{\sigma - 1} \frac{\frac{1}{\sigma} A^{\sigma} \gamma z_m^{\gamma}}{\sigma - \gamma - 1} \left(\overline{z}^{FB}\right)^{\sigma-\gamma-1}$$

Together this implies that the aggregate income of all firms in each country is

$$I = (1+r) \left[\frac{\gamma n_m}{\gamma - 1} - \left(\frac{z_m}{\underline{z}^{FB}}\right)^{\gamma} f^d - \left(\frac{z_m}{\overline{z}^{FB}}\right)^{\gamma} f^x\right] - \left[\frac{1 - 1/\sigma}{\delta + r}\right]^{\sigma - 1} \frac{\frac{1}{\sigma} A^{\sigma} \gamma z_m^{\gamma}}{\sigma - \gamma - 1} \left[\left(\underline{z}^{FB}\right)^{\sigma-\gamma-1} + \left(\overline{z}^{FB}\right)^{\sigma-\gamma-1}\right] \tag{25}$$

The aggregate price P consists of prices of firms that supply their goods to the market, including both domestic and foreign firms. Since we have symmetric countries

$$P = \left[\int_n \int_{\underline{z}^{FB}}^{\overline{z}^{FB}} \overline{p}^d(z)^{1-\sigma} g(z) g(n) dz dn + \int_n \int_{\overline{z}^{FB}} \overline{p}^d(z)^{1-\sigma} g(z) g(n) dz dn + \int_n \int_{\overline{z}^{FB}} \overline{p}^f(z)^{1-\sigma} g(z) g(n) dz dn\right]^{\frac{1}{1-\sigma}}$$

where

$$\int_n \int_{\underline{z}^{FB}}^{\overline{z}^{FB}} \overline{p}^d(z)^{1-\sigma} g(z)g(n)dzdn = \left[\frac{\delta + r}{1 - 1/\sigma}\right]^{1-\sigma} \frac{\gamma z_m^\gamma}{\sigma - \gamma - 1} \left[\left(\overline{z}^{FB}\right)^{\sigma-\gamma-1} - \left(\underline{z}^{FB}\right)^{\sigma-\gamma-1}\right]$$

$$\text{and} \int_n \int_{\overline{z}^{FB}} \overline{p}^d(z)^{1-\sigma} g(z)g(n)dzdn = \int_n \int_{\overline{z}^{FB}} \overline{p}^f(z)^{1-\sigma} g(z)g(n)dzdn = \left[\frac{\delta + r}{1 - 1/\sigma}\right]^{1-\sigma} \frac{\gamma z_m^\gamma}{1 - \sigma + \gamma} \left(\overline{z}^{FB}\right)^{\sigma-\gamma-1}$$

Thus, we have

$$P = \left[\left[\frac{\delta + r}{(1 - 1/\sigma)}\right]^{1-\sigma} \frac{\gamma z_m^\gamma}{1 - \sigma + \gamma} \left[\left(\underline{z}^{FB}\right)^{\sigma-\gamma-1} + \left(\overline{z}^{FB}\right)^{\sigma-\gamma-1}\right]\right]^{\frac{1}{1-\sigma}} \tag{26}$$

Using (25) and (26) along with (22) and (23) in the definition of A, we have

$$A = \sigma^{1/\sigma} \left\{ \left[1 - \left[\frac{1 - 1/\sigma}{\gamma}\right]\right] \times \frac{\left[\frac{\delta+r}{1-1/\sigma}\right]^{-\gamma} \frac{\gamma z_m^\gamma}{1-\sigma+\gamma} \left[\left[f^d\right]^{(\sigma-\gamma-1)/(\sigma-1)} + \left[f^x\right]^{(\sigma-\gamma-1)/(\sigma-1)}\right] (1+r)^{-\gamma/(\sigma-1)}}{N} \right\}^{\frac{-(\sigma-1)}{\gamma\sigma}}$$

Then, substitute this into the (22) and (23) we have the cutoffs value stated in Proposition 1 and Assumption 3 will guarantee that $z_m < \underline{z}^{FB}$. Q.E.D.

Proof of Lemma 1

Let ν^d be the lagrange multiplier on the feasibility constraint and λ^d be the lagrange multiplier on the borrowing constraint. Then the necessary and sufficient conditions for a solution to the the constrained maximization problem are

$$R_1^d(i + b, z) + (1 - \delta) - (1 + r) - \nu^d \leq 0, \; i \geq 0 \tag{27}$$

$$R_1^d(i + b, z) + (1 - \delta) - (1 + r) - \lambda^d \leq 0, \; b \geq 0 \tag{28}$$

$$\nu^d [n - i] = 0, \; \nu^d \geq 0 \tag{29}$$

$$\lambda^d [\eta n - b] = 0, \; \lambda^d \geq 0 \tag{30}$$

We prove by ruling out different cases to arrive at the solution that domestic firms with $z > \underline{z}$ such that $0 < i^d(n; z) + b^d(n, z) < \overline{k}^d(z)$, must have both $i^d(n, z) > 0$ and $b^d(n, z) > 0$. Thus, in this case both constraints must be binding i.e. $i^d(n, z) = n$ and $b^d(n, z) = \eta n$ and, as a result, the level of production is $k^d(n, z) = (1 + \eta)n$. Since $k^d(n, z) < \overline{k}^d(z)$, this is the case where $n < \frac{\overline{k}^d(z)}{(1+\eta)}$. Using similar technique, we can also show that for those firms such that $z > \overline{z}$ with $0 < i^x(n, z) + b^x(n, z) < \overline{k}^x(z)$, both constraints will be binding as well. Thus, these are the firms with $k^x(n,z) = (1+\eta)n < \overline{k}^x(z)$, and $n < \frac{\overline{k}^x(z)}{(1+\eta)}$. In other words, with $n \geq \frac{\overline{k}^d(z)}{(1+\eta)}$, domestic firms can attain the unconstrained scale of production $k^d(n,z) = \overline{k}^d(z)$ while exporting firms with $n \geq \frac{\overline{k}^x(z)}{(1+\eta)}$, will be unconstrained and can finance $k^x(n,z) = \overline{k}^x(z)$. Thus, for constrained domestic firms with $n \in [\underline{n}^d(z), \overline{n}^d(z))$, we have $\pi^d(n;z) = R^d((1+\eta)\,n, z) + (1-\delta)\left[(1+\eta)\,n - f^d\right] - (1+r)\,(1+\eta)\,n + (1+r)n$. Given this functional form, $\pi_1^d(n;z) = (1+\eta)\left[R_1^d((1+\eta)\,n,z) + (1-\delta) - (1+r)\right] + (1+r)$ and $\pi_{11}^d(n;z) = (1+\eta)^2\,R_{11}^d((1+\eta)\,n)$. Since $k^d(n,z) < \overline{k}^d(z)$, $R_1^d((1+\eta)\,n,z) + (1-\delta) - (1+r) > 0$ and, thus, $\pi_1^d(n,z) > 0$ while by the properties of the revenue function in (5) $\pi_{11}^d(n,z) < 0$. Now, we have established that the return to net worth accumulation of a constrained firm is higher than the return from the risk-free assets and this return is decreasing and approaching $(1+r)$ as the net worth approaches $\overline{n}^d(z)$. On the contrary for $n \geq \overline{n}^d(z)$, $\pi^d(n;z)$ is strictly increasing and linear in n and, in particular, $\pi_1^d(n;z) = (1+r)$. Similar proof can be used to show that similar properties hold for the case of exporting firms. In particular, for firms with $n \in [\underline{n}^x(z), \overline{n}^x(z))$ and $z \geq \overline{z}$, we have $\pi^x(n;z) = R^x((1+\eta)\,n,z) + (1-\delta)\left[(1+\eta)\,n - f^d - f^x\right] - (1+r)\,(1+\eta)\,n + (1+r)n$ and, thus, $\pi_1^x(n;z) = (1+\eta)\left[R_1^x((1+\eta)\,n,z) + (1-\delta) - (1+r)\right] + (1+r) > (1+r) > 0$ and $\pi_{11}^x(n;z) = (1+\eta)^2\,R_{11}^x((1+\eta)\,n) < 0$. Q.E.D.

Proof of Proposition 2

In the proof of Lemma 1, we have $\frac{\overline{k}^d(z)}{(1+\eta)}$ and $\frac{\overline{k}^x(z)}{(1+\eta)}$ defining the minimum level of net worth such that domestic and exporting firms will not be constrained. This gives the value of the two cutoffs $\overline{n}^d(z)$ and $\overline{n}^x(z)$. Since $\overline{k}^x(z) > \overline{k}^d(z)$ and both are strictly increasing in z as shown in the proof of Proposition 1, we have $\overline{n}^x(z) > \overline{n}^d(z)$ for all firms with $z > \overline{z}$ and both increasing in z as well. The curvature of $\overline{k}^x(z)$ and $\overline{k}^d(z)$ also implies the curvature of $\overline{n}^x(z)$ and $\overline{n}^d(z)$ and as shown earlier, it depends on the size of the elasticity of substitution σ. Before proving part (ii), first it is important to note that firms never operate if the maximum amount capital they can finance $(1+\eta)\,n$ cannot cover the fixed costs of operating f^d. Thus, those firms with $n < \frac{f^d}{1+\eta}$ will obviously never operate and this implies a lower bounded of the net worth necessary for firms to find operating profitable.

Let $W^d(i+b,z) = R^d(i+b,z) + (1-\delta)\max\left\{i+b-f^d, 0\right\} - (1+r)(i+b)$. Then, by the definition of the optimal level of production $\overline{k}^d(z)$, $W_1^d(\overline{k}^d(z),z) = R_1^d(\overline{k}^d(z),z) - (\delta+r) = 0$. Thus, for the case of the financially constrained domestic firms, i.e. those with $i^d(n,z) + b^d(n,z) < \overline{k}^d(z)$ and $\overline{k}^d(z) > 0$, we have $W_1^d(i+b,z) > 0$ and $\pi_1^d(n,z) = W_1^d((1+\eta)n,z)(1+\eta) + (1+r) > (1+r)$.

Since for $f^d \leq k \leq \overline{k}^d(z)$, $W^d(k,z)$ is continuous and strictly increasing in k, negative at $k = f^d$, and positive at $k = \overline{k}^d(z)$ for $z > \underline{z}$, by the Intermediate Value Theorem, there is a level of production $\underline{k}^d(z) > f^d$ and $\underline{k}^d(z) < \overline{k}^d(z)$ such that $W^d(\underline{k}^d(z),z) = 0$. This also implies that $W^d(k,z) > 0$ for $k > \underline{k}^d(z)$. Note that for $z = \underline{z}$, $\underline{k}^d(\underline{z})$ and $\overline{k}^d(\underline{z})$ coincide. For those z where $\underline{k}^d(z) < \overline{k}^d(z)$, the level of production of a constrained firm $\underline{k}^d(z) = (1+\eta)\,\underline{n}^d(z)$ where $\underline{n}^d(z)$ solves

$$R^d((1+\eta)\,\underline{n}^d(z),z) + (1-\delta)\left[(1+\eta)\,\underline{n}^d(z) - f^d\right] - (1+r)\,(1+\eta)\,\underline{n}^d(z) = 0. \tag{31}$$

Differentiating (31) with respect to z, we have

$$\frac{\partial \underline{n}^d(z)}{\partial z} = \frac{-R_2^d((1+\eta)\,\underline{n}^d(z),z)}{(1+\eta)\,W_1^d((1+\eta)\,\underline{n}^d(z),z)} < 0$$

which from (5), the numerator is negative while the denominator is positive as $(1+\eta)\,\underline{n}^d(z) < \overline{k}^d(z)$. Therefore, we have $\frac{\partial \underline{n}^d(z)}{\partial z} < 0$. Differentiating with respect to z further we have

$$\begin{aligned}
\frac{\partial^2 \underline{n}^d(z)}{\partial z^2} &= -(1+\eta)W_1^d((1+\eta)\underline{n}^d(z),z) \times \\
&\frac{\left[R_{21}^d((1+\eta)\,\underline{n}^d(z),z)\,(1+\eta)\,\frac{\partial \underline{n}^d(z)}{\partial z} + R_{22}^d(1+\eta)\underline{n}^d(z),z)\right]}{\left[(1+\eta)W_1^d((1+\eta)\,\underline{n}^d(z),z)\right]^2} \\
&+R_2^d((1+\eta)\,\underline{n}^d(z),z)(1+\eta) \times \\
&\frac{\left[W_{11}^d((1+\eta)\underline{n}^d(z),z)(1+\eta)\frac{\partial \underline{n}^d(z)}{\partial z} + W_{12}^d((1+\eta)\underline{n}^d(z),z)\right]}{\left[(1+\eta)W_1^d((1+\eta)\underline{n}^d(z),z)\right]^2}
\end{aligned}$$

$$\begin{aligned}
A &= \sigma^{1/\sigma}\left\{\left[1-\left[\frac{1-1/\sigma}{\gamma}\right]\right] \times \right. \\
&\left. \frac{\left[\frac{\delta+r}{1-1/\sigma}\right]^{-\gamma}\frac{\gamma z_m^\gamma}{1-\sigma+\gamma}\left[\left[f^d\right]^{(\sigma-\gamma-1)/(\sigma-1)} + \left[f^x\right]^{(\sigma-\gamma-1)/(\sigma-1)}\right](1+r)^{-\gamma/(\sigma-1)}}{N}\right\}^{\frac{-(\sigma-1)}{\gamma\sigma}}
\end{aligned}$$

Since $W_{11}^d((1+\eta)\,\underline{n}^d(z),z) = R_{11}^d((1+\eta)\,\underline{n}^d(z),z) < 0$ and $W_{12}^d((1+\eta)\,\underline{n}^d(z),z) = R_{12}^d((1+\eta)\,\underline{n}^d(z),z) > 0$, we can establish that

$$\frac{\partial^2 \underline{n}^d(z)}{\partial z^2} = -\frac{[(+)\,(-)+(-)]\,(+)}{(+)} + \frac{(+)\,[(-)\,(-)+(+)]}{(+)} > 0$$

Thus, the threshold $\underline{n}^d(z)$ is strictly decreasing and strictly convex in z.

Let $W^x(i+b,z) = R^x(i+b,z) + (1-\delta)\max\left\{i+b-f^d-f^x,0\right\} - (1+r)(i+b)$. Then, using similar arguments to the case of domestic firms, we have $W_1^x(\overline{k}^x(z),z) = R_1^x(\overline{k}^x(z),z) - (\delta+r) = 0$ while for the financially constrained exporting firms, we have $W_1^x(i+b,z) > 0$ and, in particular $\pi_1^x(n,z) = W_1^x((1+\eta)n,z)(1+\eta) + (1+r) > (1+r)$. To prove part (iv), note that the level of net worth $\underline{n}^x(z)$ must satisfy the condition $W^d(k^d(n,z),z) = W^x((1+\eta)\,n,z)$. Since $R^x(k,z)$ is more sensitive to z than $R^d(k,z)$, this is also true for the functions $W^x(k,z)$ and $W^d(k,z)$. Thus, an increase in z will lead to a strictly higher increase in $W^x(k,z)$ than $W^d(k,z)$ for each k. This implies that the threshold $\underline{n}^x(z)$ must be strictly decreasing in z. Q.E.D.

Appendix 3.B: Computational Algorithm

Given $r = 1/\beta - 1$,

- Guess I_0, P_0, $V_0(n, z; I_0, P_0, r)$ and $G_0(n, z; I_0, P_0, r)$.
- Solve for both the firm's static and dynamic policy functions $\chi^d(n, z; I_0, P_0, r)$, $\chi^x(n, z; I_0, P_0, r)$, $i^d(n, z; I_0, P_0, r)$, $i^x(n, z; I_0, P_0, r)$, $b^d(n, z; I_0, P_0, r)$, $b^x(n, z; I_0, P_0, r)$, and $n'(n, z; I_0, P_0, r)$.
- Iterate on the value function until convergence $V_\infty(n, z; I_0, P_0, r)$.
- Update the aggregate variables I_k, P_k, and the distribution of firms $G_k(n, z)$ that are consistent with the individual firms' decisions and the firm exit and entry transition.
- Repeat the above until convergence.

Table 1: Summary Statistics of the Final Dataset

	Brazil	Chile
Year of the Data	2002	2003
Sample Size	1365	650
Mean Age (years)	18.0	30.4
(%) Total Market Value of Production/ GDP	2.7%	n.a.
(%) Total Sales/ GDP	2.3%	3.7%
(%) Exporters	16.0%	27.1%
(%) Loans requiring collateral	67.4%	68.5%
Value of collateral needed for a loan (% of the loan amount)	123.1%	112.6%
Mean Age Started Exporting (years)	15.2	14.1

Table 2: Access to Finance as Constraints to Investment

Brazil: Business constraint: access to finance (e.g. collateral)	Freq.	Percent
No obstacle	198	14.7
Minor obstacle	118	8.8
Moderate obstacle	230	17.1
Major obstacle	341	25.3
Very Severe Obstacle	459	34.1
Total	1346	100.0
Chile: Business constraint: access to finance (e.g. collateral)	Freq.	Percent
No obstacle	396	61.3
Minor obstacle	43	6.7
Moderate obstacle	73	11.3
Major obstacle	70	10.8
Very Severe Obstacle	64	9.9
Total	646	100.0

Table 3: Costs of Finance as Constraints to Investment

Brazil: Business constraint: costs of finance (e.g. interest rates)	Freq.	Percent
No obstacle	74	5.5
Minor obstacle	45	3.3
Moderate obstacle	108	8.0
Major obstacle	362	26.8
Very Severe Obstacle	761	56.4
Total	1350	100.0
Chile: Business constraint: costs of finance (e.g. interest rates)	Freq.	Percent
No obstacle	378	58.3
Minor obstacle	54	8.3
Moderate obstacle	107	16.5
Major obstacle	64	9.9
Very Severe Obstacle	45	7.0
Total	648	100.0

Table 4: Mean Age to Start Exporting (Years)

	Brazil	Chile
Overall	15.24	14.05
Firms with highest two-quintiles of net worth	9.17	10.69
Firms with lowest two-quintiles of net worth	16.43	15.92
Firms reported access to finance as no or minor obstacle	14.16	13.65
Firms reported access to finance as major or severe obstacle	16.14	18.94

Figure 1: Brazil's Ratios of Exporters: Asset Productivity and Net Worth

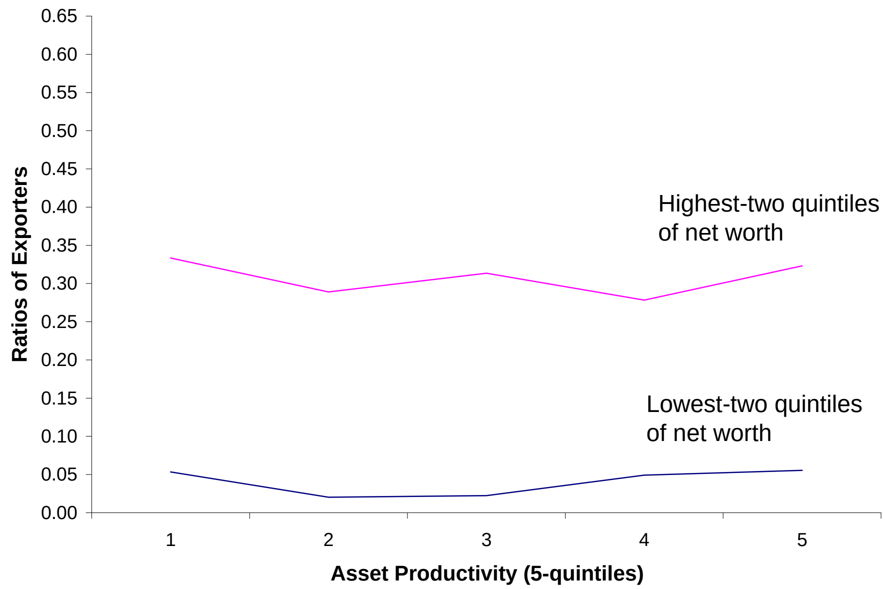


Figure 2: Chiles Ratios of Exporters: Asset Productivity and Net Worth

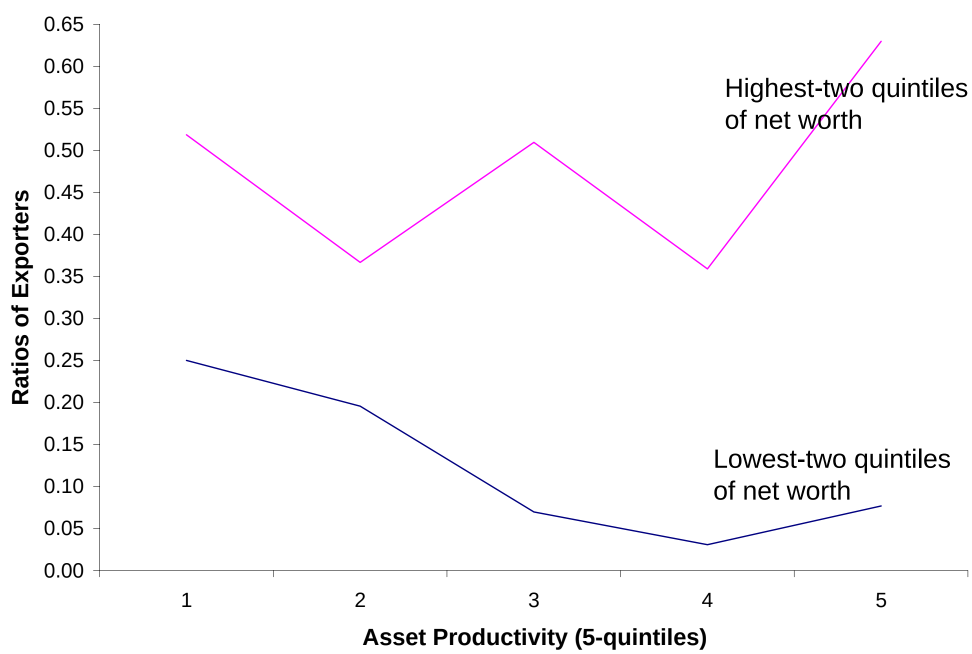


Figure 3: Brazil's Ratios of Exporters: Labor Productivity and Net Worth

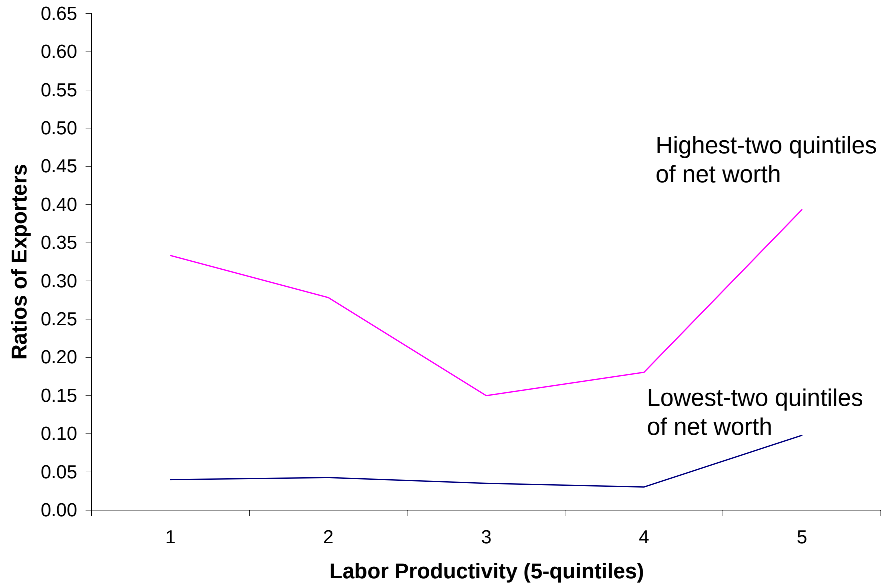


Figure 4: Chile's Ratios of Exporters: Labor Productivity and Net Worth

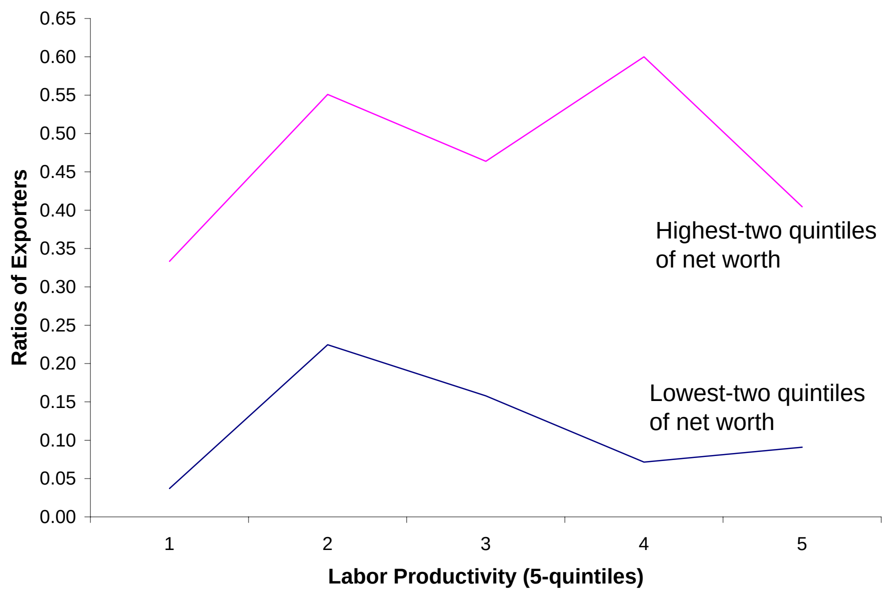


Figure 5: Brazil's Ratios of Exporters: Asset Productivity and Access to Finance

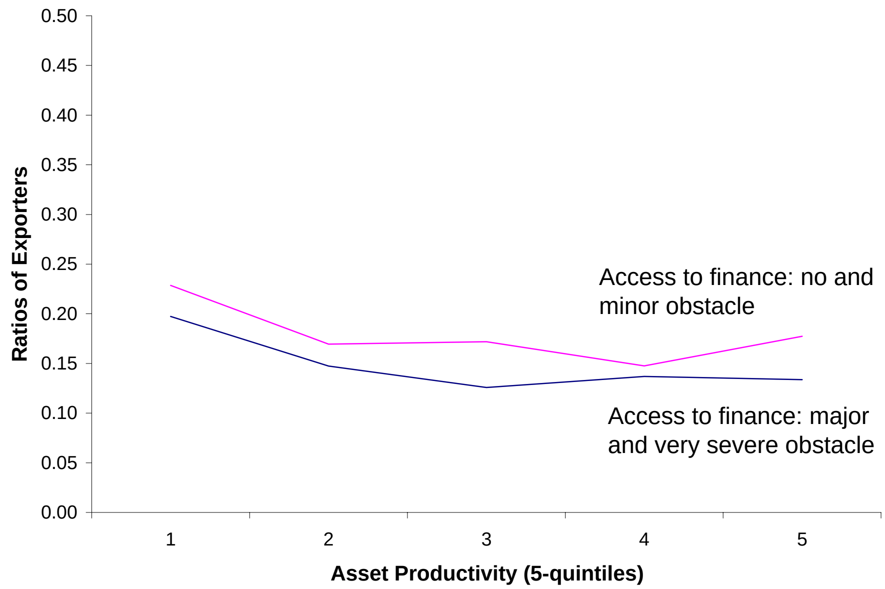


Figure 6: Chile's Ratios of Exporters: Asset Productivity and Access to Finance

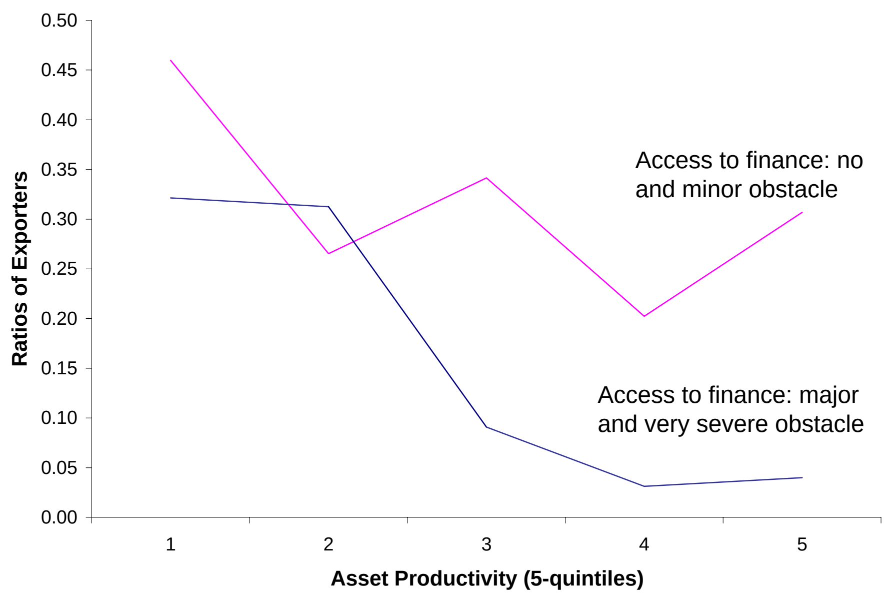


Figure 7: Brazil's Ratios of Exporters: Labor Productivity and Access to Finance

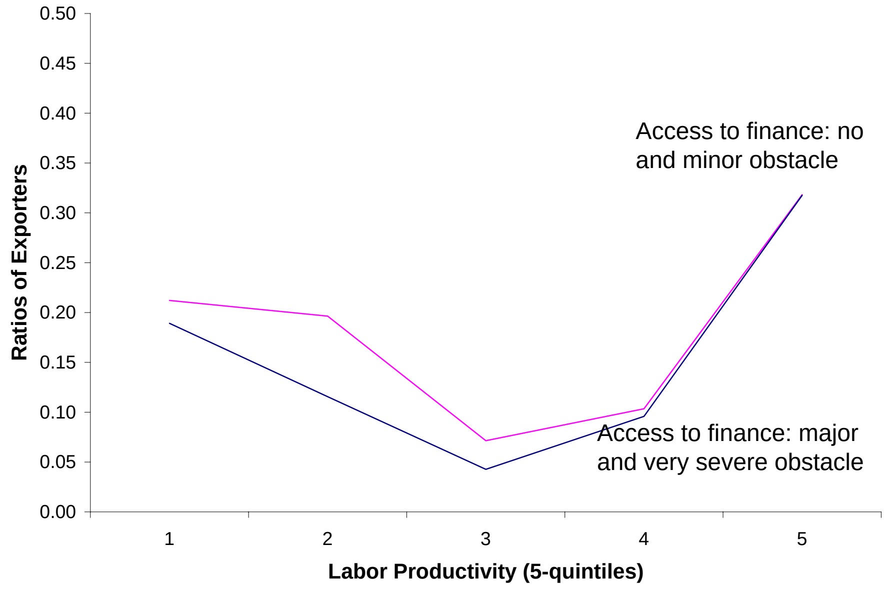


Figure 8: Chile's Ratios of Exporters: Labor Productivity and Access to Finance

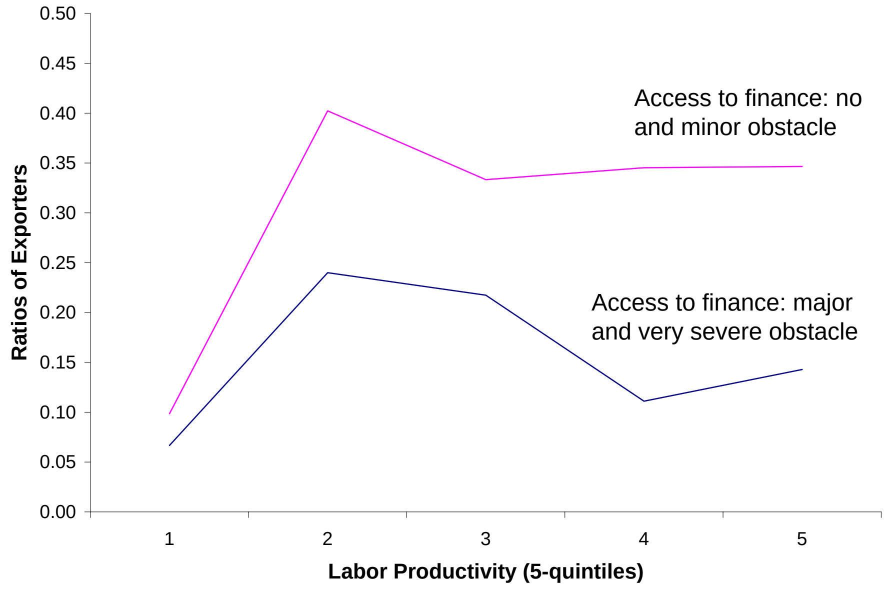
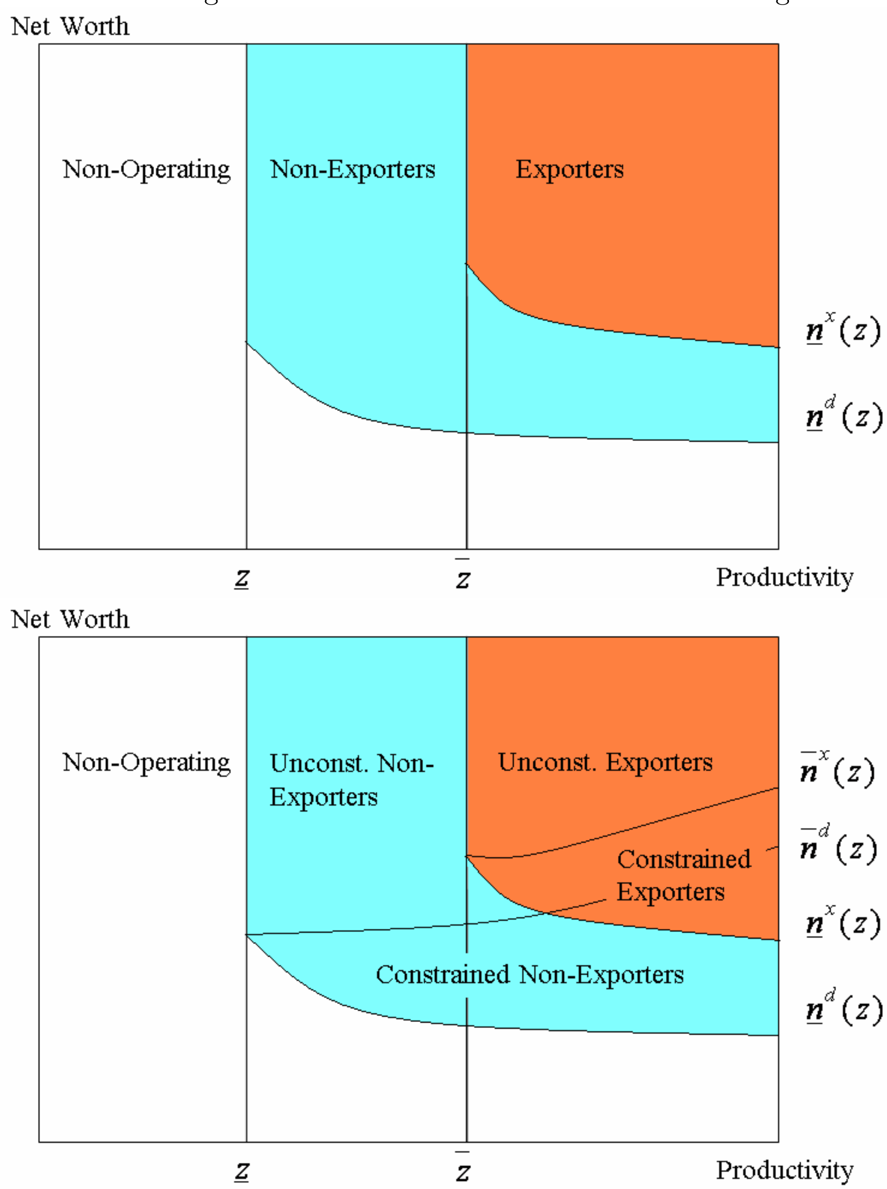


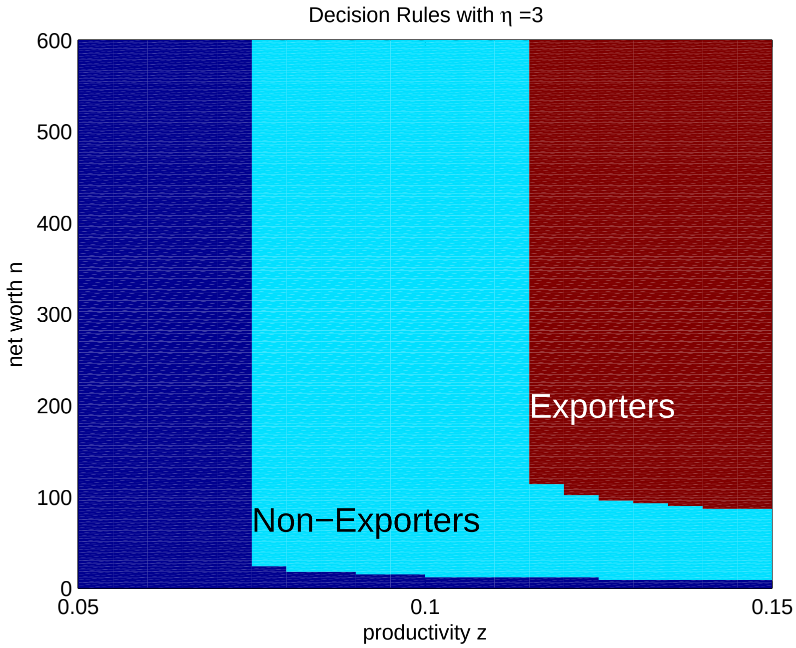
Figure 9: Firms' Decision Rules with Borrowing Constraints, $\sigma > 2$.



If the productivity cutoffs ($\underline{z}$ and $\overline{z}$) are binding, there are five groups of firms in equilibrium: non-operating firms, unconstrained and constrained non-exporting firms, and unconstrained and constrained exporting firms. Note that these productivity cutoffs ($\underline{z}$ and $\overline{z}$) may be different from the corresponding unconstrained first-best levels ($\underline{z}^{FB}$ and $\overline{z}^{FB}$).

Figure 10: Numerical Results: Firms' Entry and Exporting Decision Rules

a) $\eta = 3$ (benchmark)



b) $\eta = 0.8$

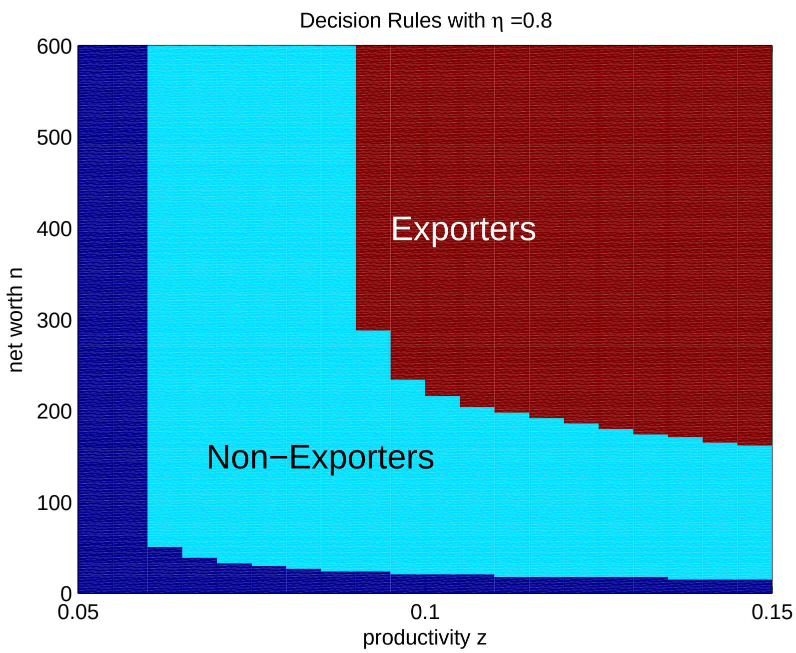
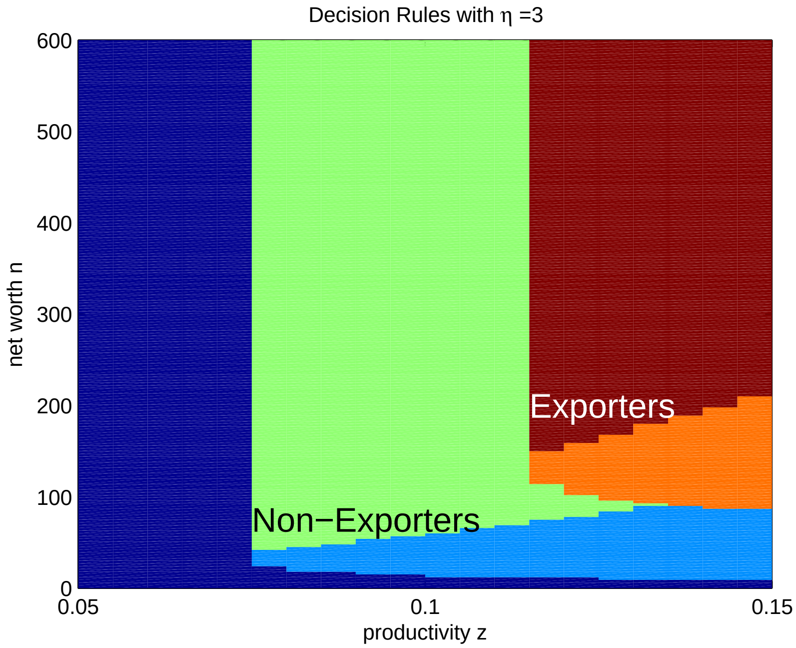


Figure 11: Numerical Results: Firms' Decision Rules

a) $\eta = 3$ (benchmark)



b) $\eta = 0.8$

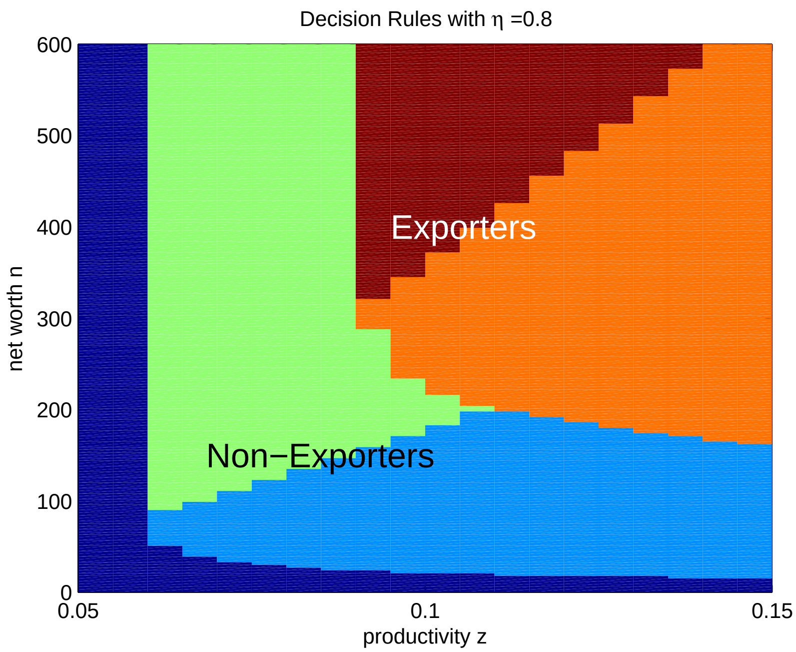
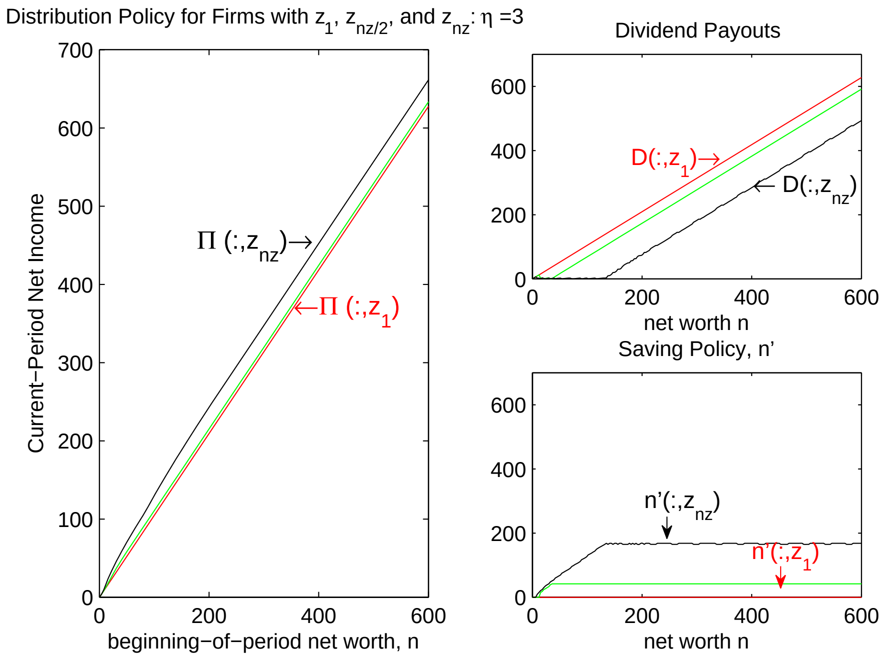


Figure 12: Firms' End-of-Period Net Worth and Dividends Policies

a) $\eta = 3$ (benchmark)



b) $\eta = 0.8$

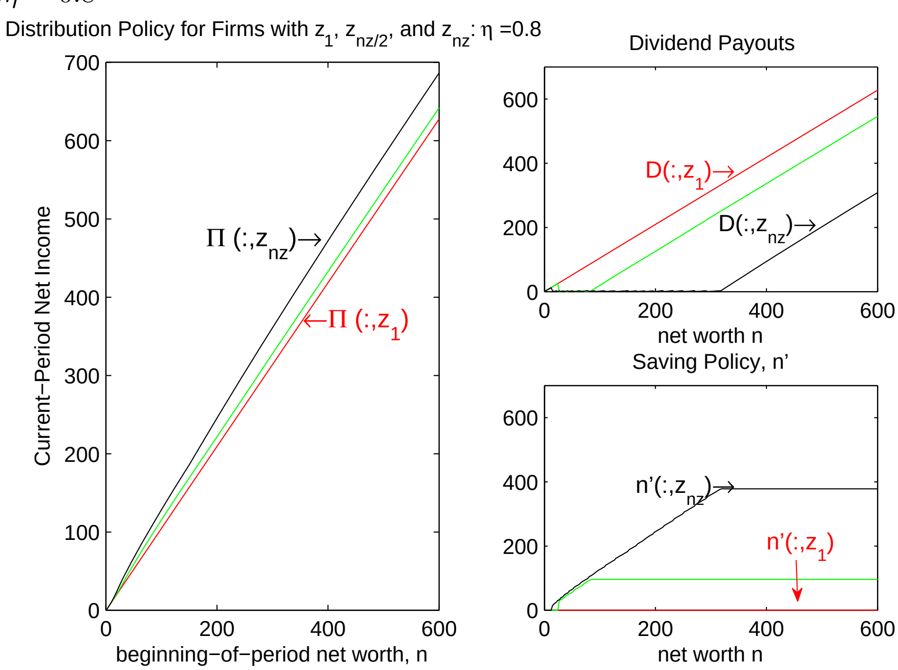
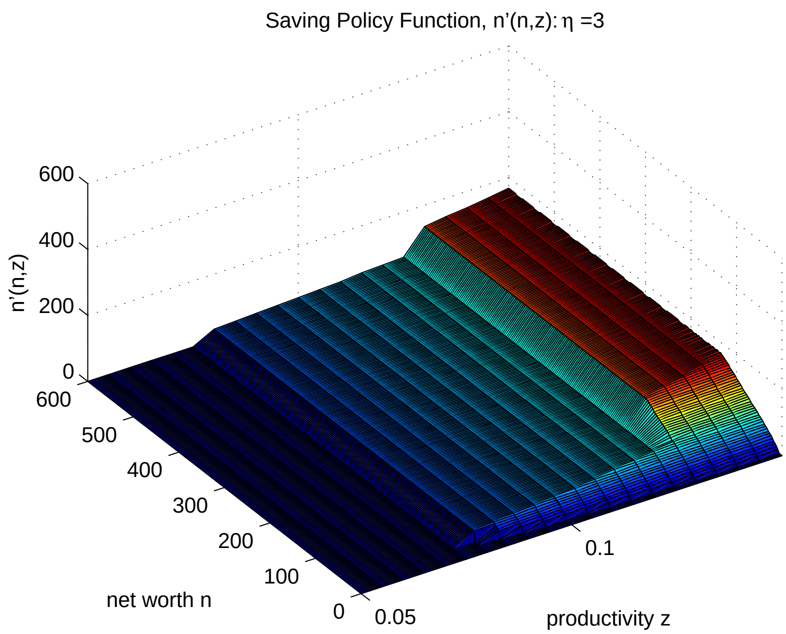


Figure 13: Firms' End-of-Period Net Worth (Saving Policy Function) over (n, z)

a) $\eta = 3$ (benchmark)



b) $\eta = 0.8$

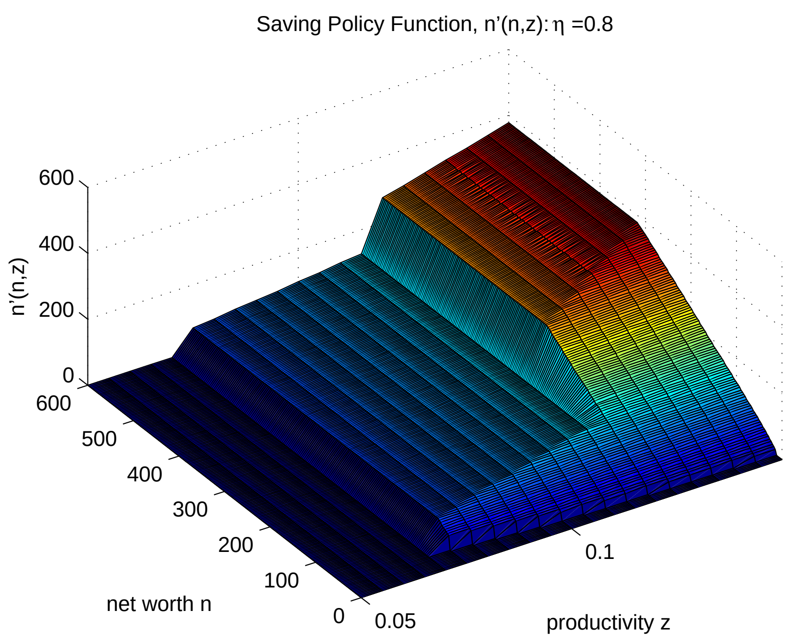
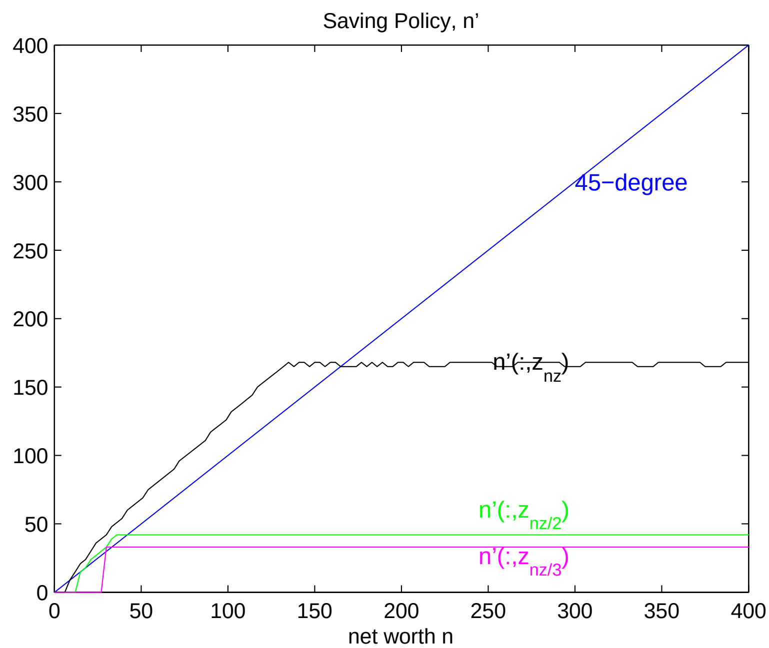


Figure 14: Saving Policy: Poverty Trap

a) $\eta = 3$ (benchmark)



b) $\eta = 0.8$